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Pricing and Policy for Carbon Capture and Sequestration with Learning by Doing

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Technical Report

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Abstract. This paper derives the efficient tax-subsidy policy in an energy-economy-environment growth model with carbon emission externalities, and a carbon capture and sequestration (CCS) sector with learning by doing (LBD) externalities. First we derive the socially optimum pricing, quantities, cashflows, and valuation. Then we derive the government tax-subsidy policies for carbon emissions and CCS that support socially efficient economic behavior with a competitive CCS industry. The Social Accounting Matrix for supporting institutional structure is derived. We analyze the qualitative dynamics of the time paths of emissions and CCS and of pricing for atmospheric carbon, sequestration capacity, and LBD.

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Pricing and Policy for Carbon Capture and Sequestration with Learning by Doing

1. Introduction

The "carbon problem" entails a complex mix of externalities.

- The deleterious climate impacts of unprecedented levels of atmospheric greenhouse gases (GHGs) affect productive activity and individual welfare without exhaustion or exclusion of effect, and are thus a *"public 'bad'"*, requiring a public process for valuation.
- Mitigation of new emission of GHGs, especially CO2, is individually costly and publicly beneficial (rival). Without institutions that exclude or charge for emissions, they use a *common pool resource*.
- Efficient carbon capture and sequestration (CCS) activity must be paid for through some agency or transaction mechanism that can compensate its social value,
- Learning by doing (LBD) while building and operating CCS capacity is not *rival* and may or may not be "appropriable", and will not be efficiently priced without intervention.[1]

When a valuable activity exhibits decreasing costs by LBD while producing, and when the LBD is a public good (non-rival and non-excludable), individual enterprises do not capture the full value of their activity. A competitive activity paid simply for its output will have too low an activity level at a market price below the social value.

When the activity in question is the capture and sequestration of carbon emissions, the socially efficient price received must be greater than the current social cost of atmospheric carbon by a "commercialization subsidy." This paper establishes the pricing and government tax-subsidy policies for carbon emissions and CCS that supports socially efficient economic behavior with a competitive CCS industry. We provide a qualitative analysis of the dynamic behavior of emissions and CCS activity and of pricing.

The economy is cast in an energy-economy-environment (E3) growth model with atmospheric carbon and CCS LBD externalities, intended to serve as a basis for development of a computable general equilibrium (CGE) model for empirical assessment of pricing and tax-subsidy policy.

The paper presents two main results:

- pricing, cashflows, values and the role of government in the efficient market economy with CCS and LBD externalities – represented in a Social Accounting Matrix for a CGE model,
- dynamics of pricing for atmospheric carbon, sequestration capacity, and LDB, and the time paths of emissions and CCS.

Government role in the efficient market economy with externalities. The general equilibrium framework here forces attention to a larger issue — a complete and consistent characterization of the role of "government" in providing a structure for the efficient internalization of costs and values of externalities in a competitive economy. We specify an institutional structure that decentralizes the socially efficient solution to a set of sectors in competitive markets in which:

- each productive sector (industrial production and CCS) is maximizing the present value of free cashflow in its market and tax-subsidy environment (the specific institutional market and incentive structure can be specified),
- the consumer sector is maximizing welfare subject to a wealth constraint consistent with its ownership in the productive sectors plus its tax-subsidy position.
- the government has a consistent balanced budget for its tax-subsidy policy,
- each asset price is the present value of asset services priced at their cost-of-carry,
- the value of each asset is the present value of its free cashflow.

The carbon emission impacts are "external" in two ways: first, if the production function aggregates over competitive individual firms, the market won't internalize the carbon emission cost without establishment of a property rights system allocating it (non-excludability); second, the impact of the cumulated carbon emission decreases aggregate production and individual welfare (non-rivalness). Both effects must be included in the carbon tax (or cap-and-trade system) on energy use. Current CCS has two benefits: current carbon stock reduction, and reduction of future CCS costs through LBD (non-rival and non-excludable, requiring the subsidy).

Thus the government/carbon sector engages in the following transactions:

- the carbon tax charge to the energy using sector comprising a charge for the cost-of-carry of matching CCS capacity plus a charge for the cost of *addition* to atmospheric carbon;
- payment to the CCS sector for the cost-of-carry of the value of the CCS capacity,
- payment to the CCS sector for the cost-of-carry of the stock of LBD offset by an equivalent charge to the consumer sector for this external benefit,
- pass-through to the consumer sector of the component of the carbon tax for the addition to atmospheric carbon.[2]

Empirical estimation of this economy's evolution will show:

- the profile of sequestration activity in the net carbon emissions balance,
- the composition of the efficient carbon price in terms of present cost and future impact,
- the role of the LBD subsidy in the price paid for sequestration, and the pace of LBD in reducing the price of sequestration activity,
- the fiscal impact of the sequestration activity and the LBD subsidy, and

- evaluation of efficiency of alternative institutional structures for the government tax-subsidy structure and the sequestration industry organization.

The complete characterization of economy's institutional structure, commodity transactions, and inter-sectoral cashflows is presented below in the Table 1: The Social Accounting Matrix for the Carbon Economy.

While the current paper aggregates fossil energy demand and treats sequestration simplisticly, disaggregation of fossil energy demand to central station, stationary distributed, and vehicular demands, with appropriate sequestration technology, cost, and LBD specifications is straight-forward.

2. LBD in the energy-environment literature

There are on the order of 5,000 GtC in the conventional fossil fuel resource base.[3] This is some 17 times the 300 GtC we have used since the beginning of the industrial revolution. Conventional fossil fuels resources — oil, natural gas, and coal — are economically convertible at current energy prices to any energy service, including transportation, and will constitute a significant share of the primary energy balance for centuries. CCS will be an essential component of the portfolio of measures required to limit the accumulation of atmospheric GHGs to an acceptable level.[4]

Either central station or atmospheric CCS, when successfully deployed, will comprise large scale material conversion processes requiring as yet undeveloped clever catalysis and process engineering.[5] Currently such deployment would be very costly.[6] However, the improvement of such technology is what human industrial activity has been good at over the last few centuries. It seems highly likely that development and deployment of CCS technologies will engender the kind of cost reductions associated with cumulated production through the "Horndahl" or "learning by doing" effect, which were famously found in the eponymous Swedish mine and the airframe industry, and which have become a standard component of facility cost projections in military and industrial costing.[7]

The LBD literature has two main strands. The first, not pursued here, is what might be called the macro-growth literature. Founded by Arrow (1962),[8] it seeks to explain how Solow-type economies can continue to grow in the face of diminishing returns to physical capital. In the energy-environmental literature in this vein,[9] it shows how induced technical change in general production can lower the efficient carbon price path. That is, it operates on the energy demand side. This analysis has gained significant micro-economic structure with the introduction by Romer of a monopolistic competitive firm structure with appropriability over embedded innovation,[10] and with the introduction of stochastic technical change and "creative destruction" through obsolescence in Aghion & Howitt (1992).[11]

A second strand is the sector-specific "experience curve" literature.[12] The treatment in this paper is very much in this spirit. We are interested specifically in the value of the stock of knowledge and the time path of its optimal subsidy relative to the cost of atmospheric carbon, and in the timing of the development of CCS capacity relative to the atmospheric carbon stock.[13] We introduce decreasing cost with LBD for investment in CCS capacity, and embed it in a standard energy-environment-economy growth model.[14]

In the context of this literature, the explicit representation of the CCS sector yields three important insights:

- It provides a sharp characterization of the optimal carbon tax.
- It provides an explicit characterization of the government tax-subsidy policies required to support an efficient market solution in the face of the environmental and LBD externalities.
- It provides a characterization of the timing of carbon emissions and sequestration activity, and of carbon and sequestration pricing.

In the remainder of the paper, we develop

- the structure of the E3 model,
- the social optimum growth structure and the carbon regime,
- the institutional and economic structure of the efficient market-government economy,
- dynamic behavior of asset prices and quantities,
- capping the atmospheric carbon overshoot,
- conclusion.

3. Structure of the Energy-Environment-Economy (E3)

The E3 economy is a Ramsey growth model expanded to represent:

- the impact of cumulated atmospheric carbon on production output and directly on utility,
- the consumption of fossil fuels in production,
- the allocation of production to consumption, capital investment, and CCS investment,
- the price of CCS capacity investment, which decreases with the stock of knowledge from cumulated investment.

There are four assets (state-variables) with associated growth equations: capital (K), atmospheric carbon (Z), CCS capacity (S), and cumulated learning (N). There are four control variables: aggregate consumption (C), energy input (E), capital investment (I^K), and CCS investment (I^S), chosen from the class of piece-wise continuous functions, $\mathcal{C}^0$, on $\mathcal{R}_+$.

The economy is characterized by four twice-continuously differentiable ($\mathcal{C}^2$) functions defined on the real non-negative orthant: the per-capita utility function, the atmospheric carbon damage function, the aggregate production function, and the CCS investment cost function.

Private and social welfare. The population/labor-force is $[L_t]_0^\infty$, with $\lim_{t\to\infty} L_t = \bar{L}$. Social welfare is:

$$\mathcal{W}_0 \triangleq \mathcal{V}_0\{L\ U(C/L, Q); \rho\},$$

where $\rho > 0$ and per-capital utility, $U(c, Q)(\in \mathcal{C}^2) : \mathcal{R}_+ \to \mathcal{R}$, is increasing and strictly concave. Per-capita consumption, $c \triangleq C/L$, and environmental quality, Q, are essential in the sense that $\lim_{c\to 0} U^{'}(c, Q) = \lim_{Q\to 0} U^{'}(c, Q) = \infty$. Environmental quality, Q, is the the negative of atmospheric carbon, $Q \triangleq \tilde{Z} - Z$, where $\tilde{Z}$ is an upper limit beyond which utility (and life) are not defined. Note that environmental quality is a "public" good in that its total quantity impacts per capita utility. For simplicity, we assume $U_{cQ} = 0$.

The interest rate is denoted:

$$r \triangleq -\frac{\mathrm{d}\ \ln e^{-\rho t}\ U^{'}(c)}{\mathrm{d}t} = \rho + \eta\,\gamma^c\ ,$$

where:

$$\eta \triangleq -\frac{c\ U''}{U'} > 0, \quad \gamma^c \triangleq \frac{\dot{c}}{c}.$$

When, in the usual growth case, $\gamma^c > 0$, then $r > \rho$.[15] Cross-scenario welfare comparisons can be measured by $W^{(1/1-\eta)}$, which is linearly homogeneous in the level of the consumption path, and which is an exact index of the Hicksian equivalent wealth variation. That is, a 1% increase in the index between scenarios has the same welfare impact as a 1% increase in wealth in the base scenario.

The atmospheric carbon output damage function. Cumulated atmospheric carbon, net of a base concentration, is Z. For simplicity, the cumulated stock of atmospheric carbon is assumed to reduce aggregate production in proportion to potential output output, $Y = F(K, E; L)$, according to a strictly convex function $\Psi(Z) : \mathcal{R}_+ \to \mathcal{R}_+$, $\Psi^{'} > 0$, $\Psi^{''} > 0$, so that the aggregate decrement to production is $Y\ \Psi(Z)$.

Atmospheric carbon stock evolution increases with an absorption coefficient, β, times carbon emissions from energy use, αE, minus sequestration, S, and decreases in proportion δ^Z to the stock itself:

$$\dot{Z} = \beta\,(\,\alpha\ E - S\,) - \delta^Z\ Z, \qquad Z(0) = 0$$

where δ^Z is the rate of atmospheric carbon disappearance into the ocean and biosphere, assumed constant.[16]

The total per capita marginal impact of Z is summarized in:

$$\Phi(Z; y, c) \triangleq U_Q(\tilde{Z} - Z)/U_c(c) + y\ \Psi^{'}(Z),$$

where $y \triangleq Y/L$.

The production function and the output constraint. Aggregate production is given by a linearly homogeneous and strictly quasi-concave function of the capital stock, the energy input, and the exogenous labor supply, $Y = F(K, E; L) : \mathcal{R}^3_+ \to R_+$, that satisfies the Inada conditions. Exogenous prices P^K and P^E, the exogenous labor supply, L, and the production function can vary with time, admitting population growth and exogenous technical change. Steady state limits are $\bar{P}^K$ and $\bar{P}^E$, and $\bar{L}$.

The unit cost of investment in capital in terms of output is P^K and unit cost of energy in terms of output is $\bar{P}^E$.[17] The output constraint is:

$$[1 - \Psi(Z)]\ F(K, E; L)\ \geq\ C\ +\ P^K I^K\ +\ P^E E\ +\ P^S(N)\ I^S.$$

Capital stock evolution is:

$$\dot{K}\ =\ I^K\ -\ \delta^K\ K, \qquad K(0)\ =\ K_0,$$

where δ^K is the rate of physical depreciation, assumed constant. For simplicity, we permit consumption out of the capital stock, so investment is not constrained to be non-negative.

With the linearly homogeneous production function, exogenous L, and $w^L \triangleq [1-\Psi(Z)]\ F_L$:

$$[1 - \Psi(Z)]\ F(K, E; L) = [1 - \Psi(Z)]\ F_K K + [1 - \Psi(Z)]\ F_E E + w^L L.$$

The CCS investment cost function. Sequestration capacity and the stock of knowledge from LBD evolve as:

$$\begin{aligned} \dot{S}\ &=\ I^S - \delta^S S, \qquad & S_0 = 0, \\ \dot{N}\ &=\ I^S, & N_0 = 0. \end{aligned}$$

The price of CCS investment is:

$$P^S(N)\ =\ \hat{P}^S + (P^S_0 - \bar{P}^S)\ n(N),$$

where $n(\in \mathcal{C}^2) : \mathcal{R}_+ \to [0, 1]$ is decreasing and strictly convex, with $n(0) = 1,\ \lim_{N\to\infty} = 0$.[18]

Significance. For the CCS problem to be substantive, we assume that in the steady state, the cost of atmospheric carbon is high enough, the value of energy is high enough, and the floor cost of CCS activity is low enough that sequestration is economic. In the steady state (denoted by "hatted" variables):

$$\begin{aligned} &Z_\infty \quad \text{solves } L\ \Phi(Z; y, c)\ =\ (\rho + \delta^S)\ \bar{P}^S, \\ &E_\infty \quad \text{solves } F_E(E, \hat{K}; \bar{L})\ =\ \bar{P}^E + \hat{\tau}^{CO_2}, \end{aligned}$$

and:

$$S_\infty\ =\ \alpha E_\infty - \delta^Z Z_\infty\ >\ 0.$$

Valuation of asset stocks and asset services flows. We summarize the standard equilibrium relationship between valuation of an asset and its asset services.

For functions $[r_t > 0]_0^\infty$, $[F_t]_0^\infty$, where $\lim_{t\to\infty}[F_t\ e^{-\int_0^t r_s\,\mathrm{d}s}] \ = \ 0$, denote:[19]

$$\mathcal{V}_t\{F;r\} \ \triangleq \ \int_t^\infty F_\tau\ e^{-\int_t^\tau r_s\,\mathrm{d}s}\ \mathrm{d}\tau, \qquad \text{so:} \quad \dot{\mathcal{V}}_t \ = \ r_t \cdot \mathcal{V}_t - F_t.$$

A stock of physical assets provides a flow of services. For an asset stock, X_t, denote the value by $\mathit{\Pi}_t^X$, and the "lease-value" of the flow of services by w_t^X.[20] Let the evolution of the asset stock and the free cashflow, FCF_t^X, from its services net of investment be:

$$\begin{aligned} \dot{X} &= I^X \ - \ \delta^X X, \\ FCF^X &= w^X X \ - \ \mathit{\Pi}^X I^X. \end{aligned}$$

Let r_t be the required rate-of-return on cashflow. Then:

$$w_t^X \ = \ \mathit{\Pi}_t^X(r_t + \delta^X) - \dot{\mathit{\Pi}}^X \quad \leftrightarrow \quad \begin{cases} \mathit{\Pi}_t^X & = \ \mathcal{V}_t\{w^X;r\}, \\ \mathit{\Pi}_t^X X_t & = \ V_t\{FCF^X;r\}. \end{cases}$$

Note that with the "own rate of interest", $r^X \triangleq r - \dot{\mathit{\Pi}}^X/\mathit{\Pi}^X$,

$$FCF^X \ = \ r^X X - \mathit{\Pi}^X \dot{X} \ = \ rX - \frac{\mathrm{d}}{\mathrm{d}t}(\mathit{\Pi}^X X).$$

4. The social optimum and the carbon regime

Optimization. The social optimization problem is:

$$\max_{(C,E,I^S\geq 0,I^K)\in\mathcal{C}^0} \ \mathcal{V}_0\ \{\ L\,U(C/L, \tilde{Z} - Z)\ ;\ \rho\ \}, \tag{1}$$

subject to:[21]

$$(\mu)\ [1 - \Psi(Z)]\ F(K,E,L) - C - P^K I^K - P^E\ E - P^S(N)\ I^S \ \geq \ 0, \tag{1-C}$$

$$(\mu\ \mathit{\Pi}^K) \qquad \dot{K} \ = \ I^K \ - \ \delta^K K\,, \qquad K_0 = \bar{K}_0, \tag{1-K}$$

$$(-\mu\ \mathit{\Pi}^Z) \qquad \dot{Z} \ = \ \beta\ (\alpha E \ - \ S) \ - \ \delta^Z Z\,, \qquad Z_0 = \bar{Z}_0, \tag{1-Z}$$

$$(\mu\ \mathit{\Pi}^S) \qquad \dot{S} \ = \ I^S \ - \ \delta^S S\ , \qquad S_0 = 0, \tag{1-S}$$

$$(\mu\ \mathit{\Pi}^N) \qquad \dot{N} \ = \ I^S\ , \qquad N_0 = 0. \tag{1-N}$$

The current value Hamiltonian is:

$$\begin{aligned} \mathcal{H} \ \ \triangleq \ \ & L\,U(C/L, \tilde{Z} - Z) \\ & + \ \mu\ \{\ [1 - \Psi(Z)]\ [\ F(K,E,L) - C - P^K\ I^K - \bar{P}^E\ E - P^S(N)\ I^S\ ] \\ & + \ \mathit{\Pi}^K\ [\ I^K - \delta^K K\ ] \ - \ \mathit{\Pi}^Z\ [\ \beta\ (\alpha E - S) - \delta^Z Z\ ] \\ & + \ \mathit{\Pi}^S[\ I^S - \delta^S S\ ] \ + \ \mathit{\Pi}^N[\ I^S\ ]\}. \end{aligned}$$

Optimality conditions are as follows. For the control variables:[22]

$$\begin{array}{lcllr}
U_c(c) & = & \mu > 0, & C > 0, & \text{(2-Cp)}\\
[1-\Psi(Z)]\ F_E(K,E;L) & = & P^E\ +\ \alpha\beta\ \Pi^Z, & E > 0, & \text{(2-Ep)}\\
P^K & = & \Pi^K\ , & I^K \text{free}, & \text{(2-Ik)}\\
P^S(N) & \geq & \Pi^S\ +\ \ \Pi^N\ \ \ \perp & I^S \geq 0, & \text{(2-Is)}
\end{array}$$

and for the state variables:

$$\begin{array}{lcllr}
[1-\Psi(Z)]\ F_K(K,E;L) & = & w^K\ \triangleq\ (r+\delta^K)\ P^K - \dot{P}^K, & & \text{(2-Kp)}\\
L\ \Phi(Z;y,c) & = & w^Z\ \triangleq\ (r+\delta^Z)\ \Pi^Z\ -\ \dot{\Pi}^Z, & & \text{(2-Zp)}\\
\beta\ \Pi^Z & = & w^S\ \triangleq\ (r+\delta^S)\ \Pi^S\ -\ \dot{\Pi}^S, & & \text{(2-Sp)}\\
-\,P^{S'}(N)\ I^S\ =\ -\dot{P}^S & = & w^N\ \triangleq\ r\ \Pi^N\ -\dot{\Pi}^N. & & \text{(2-Np)}
\end{array}$$

where the conditions on C and E follow from essentiality, and $r \triangleq \rho + \eta\ \gamma^c$. Since I^K is free, $\Pi^K\ =\ P^K$. The transversality conditions require terminal asset values to go to zero.

Asset prices are interpreted as follows:

$$\begin{aligned}
P_t^K\ &=\ \mathcal{V}_t\{w^K = [1-\Psi(Z)]\ F_K; r+\delta^K\},\\
\Pi_t^Z\ &=\ \mathcal{V}_t\{w^Z = L\Phi(Z;y,c); r+\delta^Z\},\\
\Pi_t^S\ &=\ \mathcal{V}_t\{w^S = \beta\ \Pi^Z = \tau^{CO_2}; r+\delta^S\},\\
&=\ \mathcal{V}_t\{\beta\ \mathcal{V}_t\ \{L\Phi(Z;y,c); r+\delta^Z\}; r+\delta^S\},\\
\Pi_t^N\ &=\ \mathcal{V}_t\{w^N = -\dot{P}^S(N); r\}.
\end{aligned}$$

The interpretations are:

- the value of a unit of capital, P^K, equals the discounted and depreciated value of the marginal product of capital;
- the cost of a unit of atmospheric carbon, Π^Z, equals its discounted and depreciated marginal impact on production plus utility;
- the value of a unit of CCS capacity, Π^S, is the cost of offset atmospheric carbon, which is equal to the carbon tax rate;
- the value of a unit of LBD, Π^N, is the discounted value of the induced cost reduction in $P^S(N)$.

The carbon pricing regime. The carbon tax rate can now be show to be:

$$\tau^{CO_2}\ \ \triangleq\ \ \beta\ \Pi^Z\ \ =\ \ w^S\ \ =\ \ \beta\mathcal{V}_t\{w^Z = L\Phi(Z;y,c); r+\delta^Z\}.$$

Thus the carbon tax charges for the induced damage, $\beta\ \Pi^Z$, on the cost side, and compensates on the value side for CCS service, w^S. This follows by noting that equation (2-Ep) sets the net marginal product of energy equal to its cost of production, P^E, plus the carbon tax rate, $\beta\ \Pi^Z$ times the fossil energy emission factor, α, and equation (2-Sp) sets $\beta\ \Pi^Z = w^S$.

Further, let $I^Z \triangleq \dot{Z} + \delta^Z Z$, and note that equation (1-Z) says:

$$\beta \Pi^Z \alpha E \ = \ \beta \ \Pi^Z S + \Pi^Z I^Z,$$

so that the carbon tax take pays for the current level of CCS service plus the cost of the gross addition to the atmospheric carbon stock:

$$\tau^{CO_2} \alpha E \ = \ w^S S + \Pi^Z I^Z \ = \ w^S S + w^Z Z - FCF^Z,$$

where the flow of "damage" from the atmospheric stock of carbon equals the damage to production and consumers:

$$w^Z Z \ = \ L \ \Phi(Z, y, c) Z.$$

Note that the "public bad" character of Z is reflected in the fact that the flow $w^Z Z$ is the product of two extensive variable times the per capita damage, $L \ Z \ \Phi$.

Equation (2-Is) says don't invest in new CCS capacity until its unit value, Π^S, plus the value of the LBD acquired, Π^N, equals the investment cost, $P^S(N)$. This is the key to the timing of sequestion investment, discussed below.

When $I^S > 0$,

$$P^S(N_t) \ = \ \Pi^S_t + \Pi^N_t \ = \ \mathcal{V}_t\{w^S; r + \delta^S\} + \mathcal{V}_t\{w^N; r\},$$

so each increment of investment can be paid for by the subsequent path of $w^S + w^N$.

5. Institutional and economic structure of the market-government economy

We now derive the transactions and asset valuation structure by which the social optimum can be supported in a market economy.

Begin with the consumption path:

$$C \ = \ F(K, E : L) - P^K K - P^E E - P^S(N) I^S,$$

substitute the optimal pricing equations:

$$\begin{aligned} C \ &= \ w^K K + (P^E + \alpha\beta \ \Pi^Z) E + w^L L - P^E E - (\Pi^S + \Pi^N) \ I^S, \\ &= \ w^L L + (w^K K - P^K I^K) + (w^S S - \Pi^S I^S) + \Pi^Z I^Z - \Pi^N I^S, \end{aligned}$$

complete the asset cashflows, add and subtract the externalities, $(w^Z Z - w^N N)$, appropriately, and collect terms:

$$\begin{aligned} C \ &= \ w^L L + w^Z Z - w^N N \\ &\qquad + (w^K K - P^K I^K) + (w^S S - \Pi^S I^S) + (w^N N - \Pi^N I^S) - (w^Z Z - \Pi^Z I^Z), \\ &= \ w^L L + w^Z Z - w^N N + FCF^K + FCF^S + FCF^N - FCF^Z. \end{aligned}$$

Set:

$$\begin{aligned} A_t &= \mathcal{V}_t\{C; r\},, \\ &= H_t + \mathcal{V}_t\{L\Phi Z - w^N N; r\} + P_t^K K_t + \ \Pi_t^S S_t + \ \Pi_t^N N_t - \Pi_t^Z Z_t. \end{aligned}$$

This says that if the consumers' wealth suffices to support the optimal consumption path, it must comprise the value of the productive assets – capital, CCS capacity, and LBD knowledge, minus the cost of the stock of atmospheric carbon, plus the discounted value of labor services, plus the discounted value of the externalities – compensation for the cost of environmental damage borne minus payment for the value of LBD externalities generated. Note that $\dot{A} = rA - C$.

Further, this can be rearranged to provide a statement of the "green" national income and product accounts:

$$C + P^K I^K + P^S I^S - \Pi^Z I^Z = w^L L + w^K K + w^S S.$$

Gross domestic product is consumption plus capital investment plus CCS investment minus the cost of increased atmospheric carbon. Gross national income equals the payments for labor plus payments for capital services plus payments for sequestration services valued at

$$\tau^{CO_2} \triangleq \beta\, \Pi^Z = w^S.$$

The government balanced budget includes:

- the carbon tax of

  $$\Pi^Z \alpha\beta\ E = \tau^{CO_2} \alpha E = w^S S + \Pi^Z I^Z,$$

  and offsetting payments to the CCS sector of $w^S S$ and to the consumer sector of $\Pi^Z I^Z = w^Z Z - FCF^Z$.
- a payment to the CCS sector and offsetting charge to the consumer sector of $w^N N$ for providing and consuming the externality of the cost-of-carry of the stock of LBD,

This formalizes an idea that does not emerge from partial equilibrium "Pigouvian" analyses of government intervention to correct externalities. That is, when the government imposes a tax or subsidy to internalize a negative or positive externality to a sector, and the income from that sector flows to the consumer sector, the government must pay or charge the consuming sector for the externality borne or enjoyed, in order for the wealth and cashflow balances to be consistent. Current proposals to recycle the " future damage" part of carbon tax revenue make sense only if mitigation activity such as CCS investment are optimal.

The economy cashflows are summarized in the following Social Accounting Matrix (SAM). The commodity rows add to zero (indicating market balance) and the sector columns add to zero (indicating the competitive equilibrium zero profit condition), the government column adds to zero (indicating a balanced budget), and the consumer sector satisfies its budget constraint. Specification of the SAM serves to calibrate the CGE model.

	Y	K	E	S	N	Z	A	C
p^Y	$[1-\Psi(Z)]\,Y$	$-P^K I^K$	$-P^E E$	$-\Pi^S I^S$ [a]	$-\Pi^N I^S$			$-\,C$
w^K	$-w^K K$	$+w^K K$						
w^E	$-w^E E$		$+P^E E$ $+\tau^{CO_2}\ \alpha E$					
w^L	$-w^L L$						$+w^L L$	
w^S				$+w^S S$		$-\tau^{CO_2}\ S$ [b]		
Carbon tax/ subsidy			$-\tau^{CO_2}\ \alpha E$			$+\tau^{CO_2}\ \alpha E$ [c] $-w^Z Z$	$+L\Phi Z$ [d]	
LBD tax/ subsidy					$+w^N N$		$-w^N N$	
FCF		$-FCF^K$		$-FCF^S$	$-FCF^N$	$+FCF^Z$ [c]	$+FCF^{K+S+N-Z}$	
A							$-rA+\dot{A}$	$rA-\dot{A}$
$\mathcal{V}\{FCF\}$		$-P^K K$		$-\Pi^S S$	$-\Pi^N N$	$+\Pi^Z Z$	A [f]	

[a] Recall $P^S(N)I^S \ =\ (\Pi^Z + \Pi^S)I^S$.
[b] Recall $\tau^{CO_2} \ =\ \beta\Pi^Z \ =\ w^S$.
[c] Recall $\tau^{CO_2}\ \alpha E = \ w^S S + \Pi^Z I^Z$.
[d] Recall $w^Z \ =\ L\Phi$.
[e] Recall $FCF^Z \ =\ w^Z Z - \Pi^Z I^Z$.
[f] Recall $A \ =\ \mathcal{V}\{w^L L + w^Z Z - w^N N\} + p^K K + \Pi^S S + \Pi^N N - \Pi^Z Z$.

TABLE 1. The Social Accounting Matrix for the Carbon Economy

In this SAM, the climate sector, Z, collects the carbon tax,

$$\tau^{CO_2}\ \alpha E = w^S S + \Pi^Z I^Z \ = \tau^{CO_2} S + w^Z Z - FCF^Z,$$

from the energy sector and distributes $w^S S = \tau^{CO_2} S$ to the sequestration sector to pay for services, with the balance, $\Pi^Z I^Z = w^Z Z - FCF^Z = L\Phi Z - FCF^Z$, distributed to the consumer finance sector. The LBD tax, $w^N N$, is collected from consumer finance and distributed to the LBD sector.

Of course in a real institutional setting, the LBD sector, N, won't exist on its own. At least three institutional alternatives are possible:

- The S and N sectors are consolidated, pay the market price,

$$P^S(N)\ I^S\ = (\Pi^S + \Pi^N)\ I^S$$

for investment, and receive a compensating *operating* subsidy, $w^N I^S$.
- The N and Z sectors are consolidated, and the S sector receives an *investment* subsidy, $\Pi^N\ I^S$.
- The N and Z sectors are consolidated, and the sector that manufactures I^S receives an *investment* subsidy, $\Pi^N\ I^S$.

The efficient choice depends on the evaluation of the LBD component of $P^S(N)$, the locus of the learning, and the incentive effectiveness.

6. Dynamic behavior of asset prices and quantities

A general dynamic analysis, including the interaction of the physical capital structure through r and the carbon system, is too complex. In what follows, we isolate the climate system, assuming dynamic changes in r can be ignored, and that Y increases monotonically.[23]

The steady state is characterized by $r = \rho$, $L = \bar{L}$, $P^K = \bar{P}^K = \Pi^K_\infty$, $P^E = \bar{P}^E$, $P^S(N_\infty) = \bar{P}^S$, $\Phi_\infty = \Phi(Z_\infty, y_\infty, c_\infty)$, and:[24]

$$
\begin{aligned}
&[1-\Psi(Z_\infty)] && F_K(K_\infty, E_\infty; \bar{L}) \ = \ \hat{w}^K \ = \ (\rho +\ \delta^K)\ \bar{P}^K, && \text{(5-K)}\\
&[1-\Psi(Z_\infty)] && F_E(K_\infty, E_\infty; \bar{L}) \ = \ \bar{P}^E \ + \ \alpha\beta\ \bar{P}^S, && \text{(5-E)}\\
&\hat{I}^K && = \ \delta^K\ K_\infty, && \text{(5-Ik)}\\
&\hat{I}^S && = \ \delta^S\ S_\infty, && \text{(5-Is)}\\
&\beta\ S_\infty && = \ \alpha\beta\ E_\infty \ - \ \delta^Z\ Z_\infty, && \text{(5-S)}\\
&L\ \Phi(Z_\infty, y_\infty, c_\infty) && = \ w^Z_\infty \ = \ (\rho + \delta^Z)\ \Pi^Z_\infty\ , && \text{(5-Z)}\\
&\beta\ \Pi^Z_\infty && = \ w^S_\infty \ = \ (\rho + \delta^S)\ \bar{P}^S, && \text{(5-Kp)}\\
&\Pi^S_\infty && = \ \bar{P}^S, && \text{(5-Sp)}\\
&\Pi^N_\infty && = \ 0, && \text{(5-Np)}\\
&c_\infty && = \ w^L_\infty\ \bar{L} \ + \ \bar{L}\ \Phi_\infty\ Z_\infty + \ \rho\ \left[P^K\ K_\infty \ + \ \bar{P}^S\ S_\infty \ - \ \Pi^Z_\infty\ Z_\infty\right] && \text{(5-C)}\\
& && = \ [1-\Psi(Z_\infty)]\ y_\infty \ - \ \bar{P}^K\ \delta^K\ K_\infty \ - \ \bar{P}^S\ \delta^S\ S_\infty. && \text{(5-Y)}
\end{aligned}
$$

The critical dates in the evolution of the system are:

$t^{max}_{\Pi^S}$ when $\dot{\Pi}^S = 0$,

$t^{max}_{\Pi^Z}$ when $\dot{\Pi}^Z = 0$,

$t^{+}_{I^S}$ when I^S goes positive,

$t^{max}_{\Pi^N}$ when $\dot{\Pi}^N = 0$,

t^{max}_{Z} when $\dot{Z} = 0$,

∞ the terminal steady state.

We show that:

$$t^{max}_{\Pi^S} < t^{max}_{\Pi^Z} < t^{+}_{I^S} < t^{max}_{\Pi^N} , \tag{6P}$$

$$t^{+}_{I^S} < t^{max}_{Z}. \tag{6Q}$$

The stock of atmospheric carbon overshoots and then declines to its steady-state value. The qualitative behavior of S appears to be indeterminate. It may approach its steady-state value from below, or it may overshoot.

Price dynamics. Beginning early in the system's evolution, when Z is low, $I^S = 0$, and the P^S pricing constraint is slack, $\dot{\Pi}^Z > 0$ and $\dot{\Pi}^S > 0$ (if not, they would go negative),[25] and $\dot{\Pi}^N = r\Pi^N > 0$.

The crucial date is $t^{+}_{I^S}$, when I^S goes positive and CCS capacity and activity begin to build. From this time on:

$$P^S(N) = \Pi^S + \Pi^N,$$
$$\dot{P}^S(N) = \dot{\Pi}^S + \dot{\Pi}^N.$$

The first equation says that new CCS investment starts up only when its cost equals the value of CCS capacity plus the value of LBD.

Combining with equations (2-Sp) and (2-Np):

$$\dot{\Pi}^S = -r\Pi^N < 0, \tag{4-Ps}$$

$$\beta\ \Pi^Z = (r + \delta^S)\ \Pi^S + r\Pi^N = rP^S + \delta^S \Pi^S. \tag{4-Pz}$$

Equation (4-Ps) says that after the startup of I^S, $\dot{\Pi}^S < 0$. Since now Π^S is decreasing at $t^{+}_{I^S}$, by continuity its maximum must be prior to that, and it then approaches its steady state value, $\Pi^s_\infty = \bar{P}^S$, from above. That is, by continuity, $t^{max}_{\Pi^S} < t^{+}_{I^S}$, and $\Pi^{S^{max}} > \Pi^S_\infty = \bar{P}^S$.

However, at $t^{+}_{I^S}$, $\dot{\Pi}^N = r\Pi^N > 0$, but $\Pi^N \to 0$, so $t^{+}_{I^S} < t^{max}_{\Pi^N}$, and $\Pi^{N^{max}} > \Pi^N_\infty = 0$.

Equation (4-Pz) says that the value of capacity to reduce the cost of atmospheric carbon, $\beta\ \Pi^Z$, equals the cost-of-carry of CCS capacity — the interest charge on the price of new investment plus the depreciation charge on the value of CCS capacity.

Since P^S is non-increasing, and $\mathit{\Pi}^S$ is decreasing at and after $t^{max}_{\mathit{\Pi}^S} < t^+_{I^S}$, (4-Pz) says $\mathit{\Pi}^Z$ must be decreasing, and by continuity, its maximum must be prior, $t^{max}_{\mathit{\Pi}^Z} < t^+_{I^S}$, and it then approaches its steady state value from above.

However, equation (2-Sp) says that as $\dot{\mathit{\Pi}}^S$ goes negative, $\mathit{\Pi}^Z$ increases, so $t^{max}_{\mathit{\Pi}^S} < t^{max}_{\mathit{\Pi}^Z} < t^+_{I^S}$. Since at $t^+_{I^S}$, $\dot{\mathit{\Pi}}^N = r\mathit{\Pi}^N > 0$, and $\mathit{\Pi}^N_\infty = 0$, $t^+_{I^S} < t^{max}_{\mathit{\Pi}^N}$. These demonstrate (6P).

Quantity dynamics. At $t^{max}_{\mathit{\Pi}^Z}$,

$$L\ \mathit{\Phi}(Z^{max}, y, c) = (r + \delta^Z)\mathit{\Pi}^Z \quad > \quad (\rho + \delta^Z)\mathit{\Pi}^Z_\infty = \bar{L}\ \mathit{\Phi}(Z_\infty, y_\infty, c_\infty),$$

so Z has overshot its long-run steady-state value.

Further, thereafter $\mathit{\Pi}^Z$ is decreasing, so E is increasing. Because $S_\infty > 0$, it must be that $\dot{Z} > 0$ at $t^+_{I^S}$, so $t^{max}_Z > t^+_{I^S}$, after which $\dot{Z} < 0$.

Then $\beta S = \alpha\beta\ E - \delta^Z Z - \dot{Z}$, and with E increasing, $-\delta^Z Z$ increasing and $-\dot{Z}$ positive but eventually decreasing, it appears that S may continue to grow, reaching the steady state from below. It may overshoot, driving $\dot{Z}$ sharply negative, and then retreat to its steady-state value.

7. Capping atmospheric carbon

The fact that the atmospheric carbon stock overshoots the long-run steady-state level may suggest that the environmental damage function may not completely capture policy goals.

A cap may be imposed on the optimization (and included in Hamiltonian):

$$(\mu\nu^Z) \qquad \bar{Z}_{max} - Z \geq 0.$$

Then the Euler condition (2-Zp) for $\dot{\mathit{\Pi}}^Z$ would be rewritten:

$$L\ \mathit{\Phi}(Z; y, c) + \nu^Z \ = \ w^Z \ = \ (r + \delta^Z)\ \mathit{\Pi}^Z \ - \dot{\mathit{\Pi}}^Z, \tag{2-Zp''}$$

so:

$$\mathit{\Pi}^Z_t \ = \ \mathcal{V}_t\{w^Z = L\mathit{\Phi}(Z; y, c) + \nu^Z; r + \delta^Z\}, \tag{2}$$
$$\mathit{\Pi}^S_t \ = \ \mathcal{V}_t\{w^S = \beta\mathit{\Pi}^Z; r + \delta^Z\}. \tag{3}$$

If the cap becomes active in the intertemporal solution, the anticipation of ν^Z raises $\tau^{CO_2} = \beta\mathit{\Pi}^Z$ high enough to choke off energy growth until sequestration capacity catches up. Higher $\mathit{\Pi}^Z$ in the future results in a higher profile for $\mathit{\Pi}^S$, initiating earlier sequestration investment.

8. Conclusion and extensions

For an economy with a CCS sector in which investment costs decline with LBD, we have characterized the optimal price and quantity behavior, and provided an institutional structure with specific set of balanced government tax-subsidy policies that supports the optimal solution in a competitive market economy.

We have, as well, provided a qualitative characterization of the dynamic behavior of this economy.

The empirical implementation from an appropriate CGE model will rely on this basis.

There are extensions with policy relevance:

- inclusion of R&D expenditure which potentiates LBD,[26]
- multiple fossil fuel demand sectors — central power stations, synfuels, stationary distributed demands, vehicle fuel — with specific mitigation technologies, costs and LBD parameters,
- endogenous technical change (LBD) on the demand side, which will lower carbon prices and "compete" for mitigation value.

Notes

[1]If not compensated, then it will be underproduced. If it is "appropriable" (a *club good* through patent ownership, say, then patent royalties will price it positively, and thus inefficiently, and it will be under produced and underutilized.

[2]This "recycles" the stock addition component of the carbon tax. (The other component goes to pay for the use of sequestration capacity.) The pass-through of the atmospheric carbon "investment" cost is shown to be equal to a credit for bearing the external cost of the marginal decrease in output minus a "capital account" charge for the present value of damage from the atmospheric stock of carbon. This proposition is clarified in the text, see footnote c to Table 1. The payment to the consumer sector for the external cost-of-carry on environmental damage from the stock of atmospheric carbon is shown to equal the output loss minus a "convexity" surplus due to the increasing marginal damage function.

[3]Rogner (1997).

[4]Lackner (2002).

[5]See Lackner, Wendt, Butt, Joyce & Sharp (1995), Goff & Lackner (1998), Lackner, Ziock & Grimes (1999), Yegulalp, Lackner & Ziock (2000), and Butt, Lackner, Wendt, Nomura & Yanagisawa (1999).

[6]See Yegulalp et al. (2000),Metz, Davidson, de Coninck, Loos & Meyer (2005).

[7]See Asher (1956) for the airframe experience, Alchian (1963) for an early discussion in the economic literature, and David (1973) for an historical application.

[8]See also Nelson (1959), Kamien & Schwartz (1968).

[9]See Nordhaus (1997), Goulder & Schneider (1999), Nakicenovic (2002) in Grubler, Nakicenovic & Nordhaus (2002), Wing (2003), Otto, Loschel & Dellink (2005).

[10]See Romer (1986), Romer (1990).

[11]See Aghion & Howitt (1998, Ch. 2, 6) for an overview. An approach with a more complex, empirical orientation is the econometric, multisectoral modeling pioneered by Jorgenson, see Jorgenson & Wilcoxen (1992).

[12]See Asher (1956), Alchian (1963), Conley (1970), International Energy Agency (2000) and for applications for photovoltaic technology, Williams & Terzian (1993), van de Zwaan & Rabl (2004), Modi (2007).

[13]Goulder & Mathai (2000) appears closest to the spirit of this analysis, but starts with an exogenously specified path of carbon emissions, rather than endogenizing its joint dependence with the net carbon price. Kverndokk, Rodendahl & Rutherford (2004*b*) and Kverndokk, Rodendahl & Rutherford (2004*a*) discuss timing and lock-in. See also Manne & Barreto (2002) and Manne & Richels (2002), Baudry (2000), Buonanno, Carraro, Castelnuovo & Galeotti (2000).

[14]See Lau, Pahlke & Rutherford (2000) and Kalvelagen (2003) for the standard Ramsey growth model implementation. For energy-environment-economy models, see Nordhaus's DICE-RICE models in Nordhaus & Boyer (2000), the Manne-Richels EPRI MERGE model Manne & Richels (1992), Pant & Fisher (2004), and the collection of models in the Stanford Energy Modeling Forum studies, Weyant (1999).

[15]This says that future consumption must be discounted for the passage of time and for the decrease in marginal utility with the growth of consumption and the decline in environmental quality. In application:

$$U(c) \triangleq \frac{c^{1-\eta}-1}{1-\eta} \ ,$$

so that $U^{'}(c) = c^{-\eta}$. Then:

$$\dot{C} = C\ [\eta^{-1}\ (r-\rho) - \gamma^{L}],$$

where γ^L is the labor and population growth rate, assumed to be zero in the limiting steady state. Note that:

$$e^{-\int_0^t r\,\mathrm{d}\tau} = e^{-\int_0^t (\rho - \dot{\mu}/\mu)\,\mathrm{d}\tau} = e^{-\rho t}\mu_t/\mu_0.$$

See Stern (2007, Ch. 1) for summary discussion of intertemporal utility and discounting. See Heal (1998) for a definitive theoretical development.

[16]Based on a literature review, the specification in Nordhaus (1994) is that $\Psi(Z)$ is quadratic and is calibrated to reduce GDP by 1.8% for a doubling of atmospheric carbon. This low impact is highly controversial. The specification is: Z = atmospheric CO_2 (ppm) - 278 (ppm); $\beta = 0.64$, net of biosphere and shallow ocean absorption; $\delta^Z = 0.008$, representing net deep ocean absorption. These specifications of the damage function and the atmospheric accumulation function are cited in Goulder & Mathai (2000, p. 19). Note, that if the steady state is to be balanced growth of inputs and outputs, then δ^Z must be zero. Representative carbon content (MtC/Quad(HHV)) for fossil fuels are reported by US EIA (2006) as: natural gas 14.45, petroleum 20.29, coal 25.37. If, for simplicity, we assume 21 MtC/quad, using 2.1 GtC/ppm, then $\alpha = 0.01$ ppm/quad emissions, and $\beta\alpha = 0.0064$ ppm/quad addition to the atmospheric carbon stock (ppm).

[17]Since the period for prospective exhaustion of the fossil fuel resource base is several times that for prospective exhaustion of the environment, to avoid another state variable, we do not keep track of resource exhaustion. The issue of the existence of an optimum solution to this kind of problem is treated in Heal (1998, Appendix).

[18]The standard power law formulation is:

$$n(N) \triangleq (N+1)^{-\sigma},$$

with the more general transition equation,

$$\dot{N} = I^S - \delta^N N, \qquad N_0 = 0.$$

Then:

$$N_t = \int_0^t e^{-\delta^N (t-\tau)}\ I^S_\tau\ \mathrm{d}\tau.$$

We ignore "forgetting", $\delta^n = 0$, to make the steady state values more intelligible.

[19]If F_t is free cashflow, then this says:

$$Income \triangleq r_t \cdot \mathcal{V}_t = FCF + \dot{\mathcal{V}}.$$

Income equals free cashflow plus net increase in asset value.

[20]The wage to capital services is analogous to the wage for labor services, w^L.

[21]Again, by taking the form $\mu\ \Pi$, state-variable prices are conveniently expressed in terms of output rather than utility. The state-price for Z, a "bad", is negative, so it shows up as a positive cost. Note the sign adjustment in the Euler condition, below.

[22]Recall the current-value Hamiltonian-Lagrangian:

$$\begin{aligned}\mathcal{H} \quad \triangleq \quad & L\,U(C/L, \tilde{Z} - Z) \\ & +\ \mu\ \{\ [1 - \Psi(Z)]\ [\ F(K, E, L) - C - P^K\ I^K - \bar{P}^E\ E - P^S(N)\ I^S\ ] \\ & +\ \Pi^K\ [\ I^K - \delta^K K\ ]\ -\ \Pi^Z\ [\ \beta\ (\alpha E - S) - \delta^Z Z\ ] \\ & +\ \Pi^S[\ I^S - \delta^S S\ ]\ +\ \Pi^N[\ I^S\ ]\}.\end{aligned}$$

Note that after eliminating I^K and I^S, the Hamiltonian is strictly concave in the control and state variables, if the Hessian of $-\Phi(Z)F(K, E, ; L)$ is negative definite. The following conditions are a necessary and sufficient characterization of the unique optimum plan if it exists: the KKT conditions with respect to the control variables:

$$\frac{\partial \mathcal{H}}{\partial C} = U_c - \mu\ \leq\ 0 \qquad \perp \quad C \geq 0\,, \tag{2-C}$$

$$\frac{\partial \mathcal{H}}{\partial I^K} = \mu\ [\ -P^K + \Pi^K\ ]\ =\ 0, \qquad I^K \text{ free}, \tag{2-K}$$

$$\frac{\partial \mathcal{H}}{\partial E} = \mu\ [\ F_E - \bar{P}^E - \alpha\ \beta\ \Pi^Z\ ]\ \leq\ 0 \qquad \perp \quad E \geq 0\,, \tag{2-E}$$

$$\frac{\partial \mathcal{H}}{\partial I^S} = \mu\ [\ -P^S(N) + \Pi^N\ + \Pi^S\ ]\ \leq\ 0 \qquad \perp \quad I^S \geq 0\,. \tag{2-D}$$

and the Euler conditions for the state and costate variables (recal $Y = F(K, E; L)$:

$$\begin{aligned} r\Pi^K - \dot{\Pi}^K &= \frac{\partial \mathcal{H}}{\partial K} = [1 - \Psi(Z)]\ Y - \delta^K P^K, \\ r\Pi^Z - \dot{\Pi}^Z &= -\frac{\partial \mathcal{H}}{\partial Z} = L\Phi(Z; y, c) - \delta^Z \Pi^Z, \\ r\Pi^S - \dot{\Pi}^S &= \frac{\partial \mathcal{H}}{\partial S} = \beta\Pi^Z - \delta^S \Pi^S, \\ r\Pi^N - \dot{\Pi}^S &= \frac{\partial \mathcal{H}}{\partial S} = -P^{S'}(N)\ I^S. \end{aligned}$$

The transversality conditions are:

$$\lim_{t\to\infty} e^{-\rho t}\,\mu\ \Pi^K\ K = 0\,, \tag{2-Kt}$$

$$\lim_{t\to\infty} e^{-\rho t}\,\mu\ \Pi^Z\ Z = 0\,, \tag{2-Zt}$$

$$\lim_{t\to\infty} e^{-\rho t}\,\mu\ \Pi^N\ N = 0\,, \tag{2-Nt}$$

$$\lim_{t\to\infty} e^{-\rho t}\,\mu\ \Pi^S\ S = 0\,. \tag{2-St}$$

The equations in the text follow. When $I^S \to \hat{I}^S > 0$, N grows linearly without bound. Since Π^N is bounded above by P_0^S, the transversality condition holds since the limit is dominated by the exponential term.

[23]The model will formally exhibit this behavior if $U(c) = c$, c free, so $\eta = 0$, $\mu = 1$, and $r = \rho$, violating the essentiality of consumption. Attention would be focus on parameter values that assure $c > 0$.

[24]The linearization technique in the consumption equation is due to Heal (1998).

[25]That is, for Π^Z and Π^S to converge, as required by the transversality conditions:

$$(r_0 + \delta^Z)^{-1}\ \Psi'(Z_0)\ <\ \Pi_0^Z\ <\ (r_0 + \delta^S)\ \Pi_0^S.$$

[26]As in Goulder & Mathai (2000).

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