Wind Power Forecasting Error Frequency Analyses for Operational Power System Studies

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Wind Power Forecasting Error Frequency Analyses for Operational Power System Studies

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***Abstract*—The examination of wind power forecasting errors is crucial for optimal unit commitment and economic dispatch of power systems with significant wind power penetrations. This scheduling process includes both renewable and nonrenewable generators, and the incorporation of wind power forecasts will become increasingly important as wind fleets constitute a larger portion of generation portfolios. This research considers the Western Wind and Solar Integration Study database of wind power forecasts and numerical actualizations. This database comprises more than 30,000 locations spread throughout the western United States, with a total wind power capacity of 960 GW. Error analyses for individual sites and for specific balancing areas are performed using the database, quantifying the fit to theoretical distributions through goodness-of-fit metrics. Insights into wind-power forecasting error distributions are established for various levels of temporal and spatial resolution, contrasts made among the frequency distribution alternatives, and recommendations put forth for harnessing the results. Empirical data are used to produce more realistic site-level forecasts than previously employed, such that higher resolution operational studies are possible. This research feeds into a larger work of renewable integration through the links wind power forecasting has with various operational issues, such as stochastic unit commitment and flexible reserve level determination.**

***Keywords*—wind forecasting, error frequency, hyperbolic distribution, operational power system**

I. Introduction

The wind power supplying energy in power systems has increased greatly throughout the previous decade. Unlike conventional thermal units, the variability and uncertainty of wind power has led to concerns about how wind power is utilized in power system operations. In the United States, there have been a number of studies of the impact of larger penetrations of wind power on system operations [1]–[7]. One of the results of these studies has been the recognition that wind power forecasting is an important technology enabling greater wind power penetration because it reduces the uncertainty of the wind power output. Because wind power forecasting plays a critical role in these integration studies, a proper statistical characterization of wind power forecasting errors assumed in operational studies is necessary to ensure accurate results.

The wind power forecasts used in operational studies are often divided into two forecast periods: day ahead and hour ahead. These correspond to the decision time frames in the production cost models used to simulate system operations, and are modeled on a generalization of the Unit Commitment and Economic Dispatch (UCED) problem. In this paradigm, variable values, such as load and wind power, are forecast the day before to ensure that slow starting thermal units are available to meet the anticipated load. One hour before the realization, the variable values are forecast again so that dispatch decisions may be made. Generally speaking, commitment decisions determine whether a unit will be online, whereas dispatch decisions fine tune the output level of units that will be online. However, some fast-starting units can be committed in the dispatch time frame, if necessary. Forecast errors in the unit commitment stage can have substantial economic consequences, if they are large enough that they cause a different commitment than would have been performed with an optimal forecast. For example, if the wind power is over-forecast by 500 MW, a cheaper, slow-starting coal unit, which would have been started if the forecast were more accurate, might need to be replaced by a more expensive fast-starting natural gas unit. However, if the forecast error is only 1 MW, the natural flexibility in the system will be able to make up the difference. For these reasons, most concern is placed on large forecast errors in the day-ahead time frame—i.e., the tails of the forecast error distribution have the greatest economic impact and there is more uncertainty in day-ahead forecasts.

This research seeks to first ascertain a methodology for fitting wind power forecasting errors at various levels of temporal and spatial resolution, then to harness empirical data for updating existing (numerical) site-level forecasts to increase their fidelity to measured phenomena. Both topics strive toward aiding higher resolution operational power system studies. The forecasting errors are examined in the sense of frequency and are not examined time sequentially; however, the updated forecasts are inherently a time series while maintaining distribution moments informed by the empirical data. The data for wind power forecasts and numerical actualizations come from the Western Wind and Solar Integration Study (WWSIS), with more than 30,000 locations spread throughout the western United States and a total wind power capacity of 960 GW.

II. Methods and Data

In this section, some background is provided on the primary statistics used to examine the forecast error distributions studied in this research. Additionally, the empirical data sets examined in this work are described. The analysis was performed using the R statistical computing environment [8], using additional pertinent packages [9], [10].

A. Some Statistical Background

Most work in wind power forecasting uses the first two standard moments, mean and variance, to describe the observed error distribution. Although these two metrics provide some important information about the distribution, further information is available from the third and fourth statistical moments: skewness and kurtosis. Skewness is a measure of the symmetry of the distribution; whereas kurtosis describes the relationship between the peak and tails of the distribution. A negative skew indicates a long left tail, with the bulk of the distribution on the right side. A leptokurtic distribution is one with a high kurtosis value; whereas a platykurtic distribution has a low kurtosis value. Observed kurtosis values are often compared to the kurtosis value of the normal distribution, which can be fully described by the first two statistical moments. This value is known as the excess kurtosis, and in what follows, the term *excess kurtosis* is used synonymously with *kurtosis*. Because particular interest is placed on the tails of the distribution, the analysis of these two moments will help enable a more accurate characterization of the error distribution.

Recent research into the aforementioned wind power forecasting and related concerns [11]–[13] has shown the inadequacy of the Gaussian distribution for fitting errors of interest. It has also hinted at the applicability of the hyperbolic distribution. Nonetheless, for the data examined herein, numerous theoretical distributions were considered, including: Cauchy, Laplace, Beta, the Generalized Lambda, as well as many appropriate piecewise distributions. In the interest of brevity, only major results and insights were included.

B. Investigated Data

The primary data set examined in this work was produced by 3TIER [14], [15], as part of the WWSIS Phase 1 [6]. Wind speeds at 100 m were simulated at 10-min intervals for the years 2004, 2005, and 2006, at more than 30,000 locations in the Western United States, using a Numerical Weather Prediction model (NWP). These values were then converted to a wind power output value, using the SCORE-lite statistical correction methodology [15], [16], based on the assumption that each location contained 10 3-MW turbines. The 10-min values were averaged into hourly wind power output from each of the sites; henceforth these values will be referred to as the "actuals." An additional run of the NWP was performed with different boundary conditions to produce day-ahead wind power forecasts for each of the sites selected. These values will be referred to as "forecasts" in what follows. It is noted that these forecasts were not subject to the same site-specific statistical processing normally applied to operational systems [14].

Although the simulated "actuals" and "forecasts" were obtained from state-of-the-art modeling techniques, it was desirable to bring empirical data into consideration. Thus, data have been analyzed from three distinct regions of the United States and results later made to bear on the "forecasts" for enhanced fidelity to reality; these more realistic forecasts are termed "updated forecasts." The updated forecasts will be used in future operational studies addressing the UCED problem from various perspectives. The following describes the empirical data utilized for these concerns.

The first data set comprises aggregated wind power output and forecasts from all of the active wind power plants in the Electric Reliability Council of Texas (ERCOT) interconnection during a 13-month period. The total wind capacity included in this data set is approximately 9,000 MW. This data set includes only day-ahead forecasts, made once a day at 16:00 the previous day. It is important to note that the day-ahead forecast is not at a consistent timescale and includes forecasts between 8 and 32 hours in advance. The second data set considered consists of a year's worth of day-ahead forecasts from the California Independent System Operator (CAISO) region. These forecasts are made at 05:30 and are valid for the following midnight-to-midnight time frame, thus they represent periods 18 to 42 hours in advance. They include forecasts for 16 different wind plants with a total capacity of approximately 940 MW. The third data set used consists of the forecasts and output from a wind plant in the Xcel Energy Colorado territory. The wind plant studied has an approximate nameplate capacity of 300 MW. This data set includes three months of data from the summer and fall seasons with hourly forecasts produced every 15 minutes for the next 72 hours.

All three data sets provide useful information. The day-ahead ERCOT and CAISO forecasts are important when performing the day-ahead unit commitment process. The Xcel forecasts provide useful information on how forecasts improve with decreasing forecasting horizon. The Xcel forecasts also show the forecast errors that can be expected during smaller operational timescales, though they lack the smoothing of a geographically diverse data set. Additionally, the Xcel data set consists of a single season, instead of at least one year for the other two data sets. It is important to note that all of the data sets use methodologically similar forecasting systems, based on NWP models with statistical post-processing, but the forecasts come from two different forecast providers. For the concerns of this article, as explained in a later section with greater detail, only statistical moments from the error distributions of the three empirical data sets were considered directly (through an interpolation approach) for producing the updated forecasts.

An overview of the analysis of the wind power forecasting data set and its updating for the needs of WWSIS Phase 2 is provided in Fig. 1. This paper provides a summary of the crucial aspects of the research methodology and some final results; a forthcoming report provides greater detail.

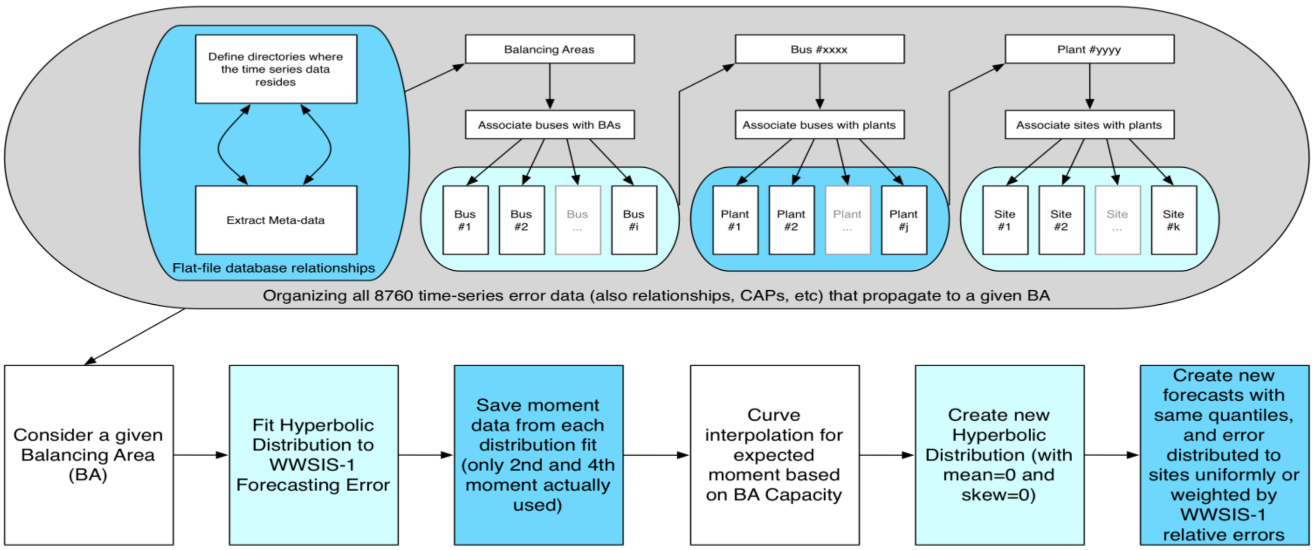

Fig. 1. Overview of wind power forecasting analysis and production of updated forecasts that consider empirical data

III. Characterization of Forecasting Errors

To understand the implications of wind power forecasting errors in wind integration studies, a statistical characterization of the errors is helpful. This characterization can identify the range of errors expected, with their corresponding frequency. Such a characterization can be particularly useful for determining the operating reserves necessary to compensate for the forecasting errors. Here, two different levels of the wind forecasting data were examined: the forecasts at individual sites and forecasts aggregated throughout balancing areas.

A. Individual Sites

The forecasts that are most commonly used by wind power developers and plant operators are those created for individual wind plants. These forecasts may be used by operators to bid into the day-ahead market in areas where wind may participate in such a market. Here, the forecast errors for a typical 30-MW wind plant in the WWSIS data set were examined. Fig. 2 illustrates the typical error frequency behavior from one of the 30,000 sites, with mean = -1.139, var = 60.4, skew = -0.206, and kurt = 2.16. The distribution is leptokurtic, and its negative bias underestimates the actual production by ~3.8% on average; the negative skew compensates in terms of the whole distribution. The assumption of normality is obviously erroneous, and the best fit from the myriad of theoretical distributions considered came from the hyperbolic distribution. The hyperbolic distribution is common in financial modeling and applications. This may be intuitive to some readers because any forecasting can be thought of as actively chasing realizations of future measurements: with a physically driven, somewhat predictable process, there will be many relatively small errors and considerably fewer relatively large errors.

B. Aggregation of Balancing Areas

Although the forecasts at individual wind plants are important for wind plant operators, the aggregate forecast for all of the wind plants is an important consideration for the Independent System Operator (ISO) or Balancing Authority (BA). Here, all of the wind plant outputs and forecasts for each of the balancing areas were aggregated according to the high-wind scenario of WWSIS Phase 2. Aggregation throughout a wide geographic domain typically creates distributions that are more conventional than from individual plants, as more atmospheric phenomena are involved, and large forecasting errors become less correlated with increasing distance. Fig. 3 provides a typical example from a BA, with mean = 137.7, var = 5.84*10^5, skew = 0.122, and kurt = 2.05. The distribution is leptokurtic, and its positive bias overestimates the actual production by ~2.6% on average; the positive skew compensates in terms of the whole distribution. It should be noted the error bias is less than that of the individual site, which would be expected according to the spatial and temporal smoothing effects previously noted. In addition to better forecasts at the larger scale because of aggregation and smoothing effects from averaging, there is a visually, and more importantly numerically, better fit for the tails of the distribution. This is fundamentally important because these extreme events drive the economics of the UCED problem and are of great interest in operational power system studies.

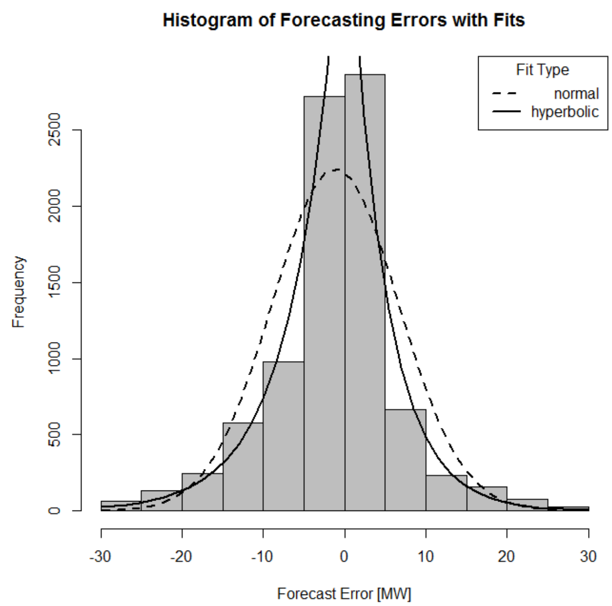

Fig. 2. The site-level forecasting errors and pdf fitting

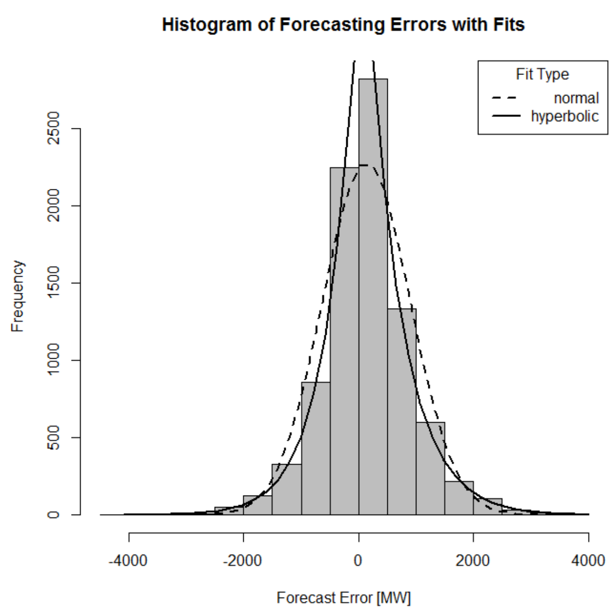

Fig. 3. The BA-level forecasting errors and pdf fitting

C. Goodness-of-Fit Evaluation

From simple visual inspection, the normal distribution is not helpful for the concerns of this research. In the interest of space, the numerous theoretical distributions considered for fitting the forecast errors were not considered here.

Focus was placed on ensuring the hyperbolic distribution met standards associated with a goodness-of-fit statistic for all BAs and a subset of selected sites of interest to the WWSIS Phase 2 high-wind scenario, after "best fits" were determined. All implementations of distribution fitting relied on a maximum likelihood routine using the Nelder-Mead simplex method for the appropriate initialization [9]. To ensure the robustness of the fit parameters, the routine was evaluated 1,000 times (for each BA and selected site) as part of a bootstrapping approach programmed by the authors, the final result being the best estimates of the four hyperbolic parameters for every BA and selected site.

The goodness-of-fit statistic utilized [17] a Cramer-von Mises statistic as follows:

$$W^2 = \sum_{i=1}^{n} \left(z_{(i)} - \frac{2i-1}{2n} \right)^2 + \frac{1}{12n}, \tag{1}$$

It has a null hypothesis that the sample comes from the hyperbolic distribution, which is accepted/rejected after four distinct calculation steps:

1) The maximum likelihood estimates are obtained;

2) The Cramer-von Mises statistic is calculated;

3) Tabular values of W^2 are consulted at significance level alpha and the null hypothesis is accepted or rejected; and

4) The p-value of the test can be calculated by interpolation, if necessary.

The null hypothesis was accepted at the alpha = 0.05 significance level for all BAs and each selected site. This validated the applicability of the hyperbolic distribution for the forecast error data under investigation for WWSIS Phase 2. It should also be noted that the hyperbolic distribution provided a superior fit at the "bus" and "plant" aggregation levels of Fig. 1 than all the theoretical distributions previously discussed and considered as part of this research.

IV. Comparison with Operational

To fully understand the implications of the simulated wind power forecasts used in wind integration studies, an understanding of how they compare with the errors observed in real operational forecasting systems is necessary; the empirical data was previously described in Section II-B. Understanding where the simulated forecast errors differ allows the identification of their limitations for use in wind integration studies. Furthermore, the empirical data can be harnessed to create more realistic forecast error distributions that help increase model fidelity of wind integration studies; the updated and more realistic forecasts will be termed the "updated forecasts," and are necessary only at the site level because of the topology and required inputs of such power system studies. Therefore, the methodology illustrated in Fig. 1 relied on extracting crucial information (i.e., statistical moments) from the empirical operational forecasting system data, then used it to produce the updated forecasts superior to those from WWSIS Phase 1 [6]. The updated forecasts are currently being used as part of various ongoing UCED investigations involving wind integration.

One of the fundamental problems with wind integration studies is that researchers are evaluating a future that does not yet exist. One of the implications of this reality is that although actual wind power forecasting errors for all of the 960 GW of installed wind capacity considered in the WWSIS would be preferred, we must create these errors, keeping in mind important features that should be represented as close to the current state as possible. With only limited empirical operational forecasting information available, it is therefore necessary to interpolate data to infer reasonable values for modifying the forecasts to produce the updated forecasts. To be able to numerically compare the important features of the error distributions, the first four statistical moments were chosen.

In the process of updating the forecasts to a more faithful representation of reality (i.e., still erroneous but not containing statistical anomalies), a key assumption made was that the bias and skew of the updated forecast error distribution should be equal to zero. That is, there was no physical reason the forecasts should, on average, over- or under-forecast, and there was no reason to expect a higher frequency of over- or under-forecast events. Therefore, in the conversion process, primary focus was placed on obtaining realistic magnitudes of variance and kurtosis, as these were the main statistical features of interest from an error frequency viewpoint. To produce the updated forecasts, the partitioning of the errors in a time series fashion was a function of the original forecast errors. This was meant to maintain the majority of the site-to-site correlations that were a product of the NWP forecast generation process. An overview of the conversion process follows in the next section.

V. Conversion Process

A visual representation of the updating process for modifying the original WWSIS Phase 1 forecasts to produce the updated forecasts is shown near the bottom of Fig. 1. The goal was to produce more realistic site-level forecasts, updating the original forecasts through the reliance on operational system data (i.e., statistical moments) and the standardization/extraction of statistical anomalies. A step-by-step process description follows.

The first step was to obtain the best hyperbolic fit to the BA-level data; examples and reasoning for this at both the BA and site levels were discussed above. However, an additional driver was the manner in which the forecasts would be used in the WWSIS Phase 2 study. The study uses a zonal transmission model, meaning that forecasts are essentially aggregated at the BA level. With the distribution fit, the four moments were saved from each BA.

The second step was to use the empirical operational data to interpolate what the moments should be (from empirical data and a curve fit) as a function of a given BA's capacity; the moments from the original forecast errors and the empirical interpolation were roughly similar. Analysis of the data revealed that the variance and kurtosis values of both the operational and original WWSIS forecast data was strongly correlated with the wind capacity considered, an indirect measure of the geographic diversity.

The third step involved determining the values of the hyperbolic parameters, with the location (mean) and asymmetry (skew) parameters assumed null, as discussed above, such that the empirical-derived operational moments could be matched in an optimal fashion. The optimization process involved a particle swarm routine programmed by the authors to evaluate a moment matching subroutine until convergence on the optimal hyperbolic parameters. The result was that the character of the original forecast errors (with hyperbolic distribution) was maintained, minus the statistical anomalies, and the empirical data concerning moments was imparted on the updated forecast error distribution. Because of histogram binning, the updated forecast error density showed only minor visual differences from the original of Fig. 3, and was therefore not provided here; however, there were distinct numerical differences. Figs. 4 and 5 illustrate the numerical differences between the WWSIS Phase 1 forecast errors and the updated theoretical ones, which took empirical data into account and were not biased or skewed. From the figures, typical site-level examples of minor and significant differences were noted, respectively. Fig. 4 shows a case where the updated forecast errors (i.e., the theoretical distribution signified by the line) were only slightly different than the original WWSIS forecast errors—mostly in the tails, where there were less than a dozen samples/hours per year; the update was minimal. Fig. 5 shows a case were the updates were significantly different than the original WWSIS forecast errors—the original forecast error data was hyperbolic but anomalous because of one sample year of hourly data that was a function of one forecasting instance, and was not indicative of realistic and quality site-level forecasts. The latter was the reason for updated forecasts, as well as time/computation limits on such minor stochastic considerations.

The fourth step involved determining the error at each hour of the year according to its quantile. This was achieved by stepping through the original WWSIS forecasts, determining its quantile at that hour, and mapping that to the quantile of the theoretical distribution to allow the determination of the updated error at each hour. The process is illustrated in Fig. 6 for one hour in the 95% quantile.

The fifth and final step was to take the 8,760 hourly errors and partition them to the site level to produce the updated forecasts. This could be accomplished in one of two ways, both of which were executed: (1) The hourly error could be uniformly distributed to each site, and/or (2) The hourly error could be distributed to each site according to its relative error, i.e., the site's hourly error as a proportion of total BA error from the original WWSIS error forecasts. The sign of the error was maintained from the original to the updated forecasts.

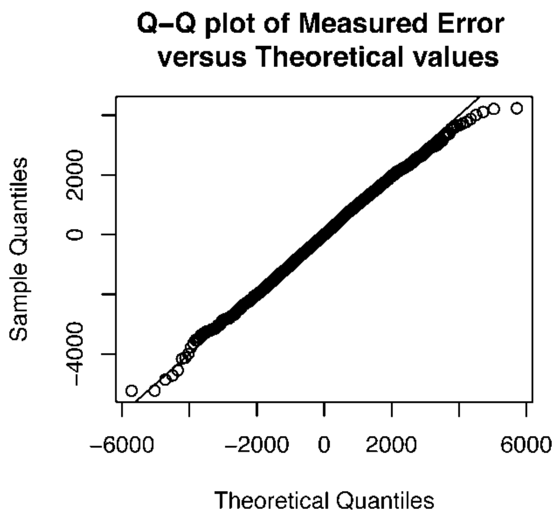

Fig. 4. Quantile-quantile plot of updated (theo.) forecast errors and the original WWSIS data: minor differences

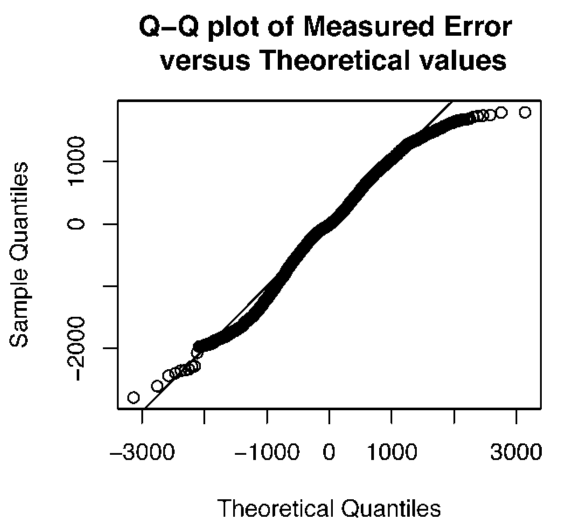

Fig. 5. Quantile-quantile plot of updated (theo.) forecast errors and the original WWSIS data: significant differences

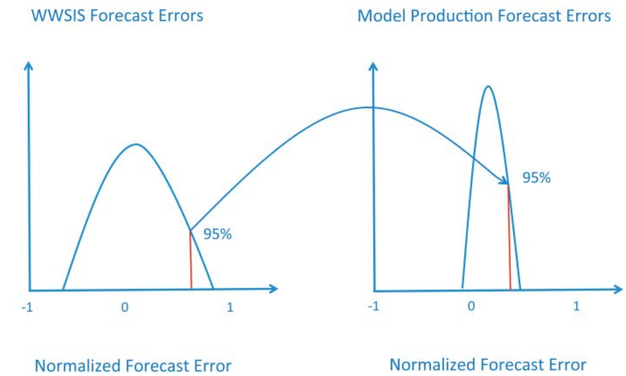

Fig. 6. The BA-level forecast errors mapped (according to quantile and sign) from the original WWSIS distribution to the updated forecast distribution

VI. Conclusions and Future Work

In this work, we compared the forecast error distributions from the WWSIS data set with those taken from operational forecasting systems. The results of the analysis enabled the modification of the forecasts from the original WWSIS data set to more accurately reflect the current state in wind power forecasting. This is important especially because there were significant differences in the tails of the error distributions, i.e., the largest forecasting errors that have the most economic impact. Sensitivity studies are planned in the WWSIS Phase 2 to quantify the economic impacts of the improved wind forecasts. The modified forecasts will be made publicly available alongside the original WWSIS data. In a broader context, this work highlights the need for accurate portrayal of wind power forecasting errors in high wind penetration integration studies. This is an often overlooked aspect of these studies that can significantly influence the results of production cost simulations. Future work will look to further disaggregate the wind power forecasting errors into different categories where similar results can be expected, e.g., based on weather conditions. This will further increase the fidelity of the wind power forecasts used in wind integration studies.

VII. Acknowledgments

The authors would like to thank Debra Lew and Greg Brinkman for their many conversations on forecasting errors in production cost models.

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