# A probabilistic framework for security constrained reserve scheduling of networks with wind power generation

Maria Vrakopoulou, Kostas Margellos, John Lygeros and Göran Andersson

*Abstract*— **This paper proposes a novel probabilistic framework to design an N-1 secure day-ahead dispatch, while determining the minimum cost reserves for power systems with high wind penetration. To achieve this, we build on previous work, and formulate a stochastic optimization program with chance constraints, which encode the probability of satisfying the transmission capacity constraints of the lines. To incorporate then a reserve decision scheme, we take into account the steady state behavior of the secondary frequency controller, and hence consider the reserves to be a linear function of the total generation-load mismatch. The overall problem results in a chance constrained bilinear program; to achieve tractability, two alternative convex reformulations are proposed, and the so called scenario approach is employed. This approach is based on sampling the uncertain parameter (in this paper the wind power) while keeping the desired probabilistic guarantees. To illustrate the effectiveness of the proposed technique we apply it to the IEEE 30-bus network, and compare the alternative reformulations in terms of cost and performance by means of Monte Carlo simulations, corresponding to different wind power realizations generated by a Markov chain based model.**

## I. Introduction

The expected increase in the installed wind power capacity highlights the necessity of revisiting certain operational concepts, like security and reserve scheduling, so as to take into account the fluctuating nature of the wind. In a liberalized power market structure both problems are tasks of the Transmission System Operator (TSO). In a deterministic setup, security of a power system refers to its ability to survive contingencies, while avoiding any undesirable disruption of service [1], [2]. As a security measure, the so called N-1 security criterion is commonly used, under which the system is considered to be N-1 secure if any single component outage does not lead to an overloaded component, or to other operational violations.

In the absence of uncertainty, many methods dealing with security enhancement have been proposed [3], [4], [5], [6], [7]. A more detailed literature review regarding such deterministic approaches can be found in [8]. In a stochastic framework though, most of the research so far has been either concentrated on the economic implications of security [9], [10], or it has resorted to Monte Carlo based statistical analysis [11], [12].

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Stochastic reserve scheduling on the other hand, has attracted significant attention since it plays an important role, both from a technical and an economical point of view. In view of maximizing the expected social welfare, optimization of reserve power has been addressed in [13], [14], [15], [16], in a security constrained market clearing context. Using the same framework, [17], [18] formulated a multi-stage stochastic program to determine the minimum cost level of reserves, modeling the uncertain generation by means of scenarios, and using reduction techniques to ensure tractability of the problem.

In this paper, we design a probabilistically robust N-1 secure day-ahead dispatch for the generating units, while determining the minimum cost reserves. In [8], we addressed the problem of generating a probabilistically secure generation dispatch, separately from the reserve scheduling one, which was tackled in [19]. To achieve both objectives, we first integrate as in [8] the security constraints, emanating from the N-1 criterion, to a DC optimal power flow problem [20], and formulate a stochastic optimization problem with chance constraints. Subsequently, modeling the steady state behavior of the secondary frequency controller, leads to representing the reserves as a linear function of the total generation-load mismatch, that may occur due to the difference between the actual wind from its forecasted value, or as an effect of a generator/load loss.

The resulting problem is a chance constrained, bilinear program. To achieve tractability, two alternative reformulations are proposed and are presented in detail in Section III. Both approaches lead to a convex problem, that can be then solved using the so called scenario approach (see Section III.C), [21]. To evaluate the efficiency of the proposed approach, both alternatives are compared in terms of cost and performance by means of Monte Carlo simulations for different wind power realizations, using a modified version of the IEEE 30-bus network, retrieved from [22].

Section II introduces the formulation of the security constrained reserve scheduling problem as a chance constrained optimization program. In Section III, two alternative convex reformulations, and details regarding the "scenario approach" are provided. Section IV illustrates the performance of the proposed approach via a simulation study, whereas Section V concludes the paper and provides directions for future work.

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## II. Problem formulation

### A. Definitions and problem set-up

For the analysis of the subsequent sections, we consider a power network comprising $N_G$ generating units, $N_L$ loads, $N_l$ lines, and $N_b$ buses. For the security constrained formulation, we take into account any outage involving the tripping of a branch, generator, or load. Denote by $N_{out} = N_G + N_L + N_l$ the total number of outages. Moreover, denote by $\mathcal{I}^l_{out}, \mathcal{I}^L_{out}, \mathcal{I}^G_{out}$ the set of indices corresponding to branch, load and generator outages, respectively.

The problem formulation of the next section is based on the following assumptions: 1) A standard DC power flow approach [23] is adopted. 2) Wind generation is located at a single bus of the network. 3) No load uncertainty is considered. The first assumption is rather standard for this type of problems, whereas the last two are included so as to simplify the presentation of our results, and could be easily captured by the proposed algorithm.

Under these assumptions, for every outage $i = 0, 1, \ldots, N_{out}$, ($i = 0$ is used for the base case) the power flow equations and the power injection vector can be written as $P^i_f = B^i_f \theta$, $P^i = B^i_{BUS} \theta$, where $P^i_f \in \mathbb{R}^{N_l}$ contains the power flows of each line, and $P^i \in \mathbb{R}^{N_b}$ and $\theta \in \mathbb{R}^{N_b}$ denote the active power injections and the voltage angles at every bus of the network respectively. Matrices $B^i_f$, which include the imaginary parts of the admittance of each network branch, and $B^i_{BUS}$, which denote the nodal admittance matrix of the network, are both topology related quantities, and hence depend on the outage $i$. For the situation that there is no outage ($i = 0$) or $i \in \mathcal{I}^L_{out} \cup \mathcal{I}^G_{out}$, $B^i_f = B_f \in \mathbb{R}^{N_l \times N_b}$ and $B^i_{BUS} = B_{BUS} \in \mathbb{R}^{N_b \times N_b}$, whereas in other cases their dimension should be changed appropriately.

For each $i = 0, 1, \ldots, N_{out}$ we eliminate $\theta$ from $P^i_f, P^i$, so as to represent the power flows $P^i_f$ as a function of the power injections $P^i$. To achieve this, since $B^i_{BUS}$ is singular with rank $N_b - 1$, we choose one angle as the reference one and set it to zero. Without loss of generality let $\theta_{N_b} = 0$. Let $\widetilde{B}^i_{BUS} \in \mathbb{R}^{(N_b-1)\times(N_b-1)}$, $\tilde{\theta} \in \mathbb{R}^{N_b-1}$, $\widetilde{P}^i \in \mathbb{R}^{N_b-1}$ denote the remaining parts of $B^i_{BUS}$, $\theta$, and $P^i$ respectively. We then have $\tilde{\theta} = (\widetilde{B}^i_{BUS})^{-1}\widetilde{P}^i$, and using $\theta = \begin{bmatrix}\tilde{\theta} & 0\end{bmatrix}^T$, we get $P^i_f = B^i_f \begin{bmatrix}(\widetilde{B}^i_{BUS})^{-1}\widetilde{P}^i & 0\end{bmatrix}^T$.

The power injection vector $\widetilde{P}^i$ can be written in a generic form as

$$\widetilde{P}^i = \Big[I^i_G C_G(P_G + R^i) + C_w P_w + I^i_L C_L P_L\Big]_{N_b-1}, \quad (1)$$

where $[\cdot]_{N_b-1}$ denotes the first $N_b - 1$ rows of the quantity inside the brackets. $P_G \in \mathbb{R}^{N_G}$, $P_w \in \mathbb{R}$, and $P_L \in \mathbb{R}^{N_L}$ denote the generation dispatch, the wind power in-feed and the load, respectively. $R^i \in \mathbb{R}^{N_G}$ is a power correction term, which is related to the reserves of each generator and will be defined in the next subsection. Matrices $C_G, C_w, C_L$ are of appropriate dimension, and their element $(i, j)$ is "1" if generator $j$ (respectively wind power/load) is connected to the bus $i$, and zero otherwise. $I^i_G \in \mathbb{R}^{N_b \times N_b}$ is an identity matrix, and based on the outage $i$, one of its diagonal elements will turn to zero, with index corresponding to the bus that the tripped generator is connected. Matrix $I^i_L \in \mathbb{R}^{N_b \times N_b}$ is defined in a similar way.

### B. Reserves representation

Reserves are needed to balance generation-load mismatches, which may occur due to a difference between the actual wind power and its forecasted value, or as an effect of a generator/load loss. Such imbalances between load and generation induce frequency deviations and activate the primary frequency controller. Secondary frequency control (or Automatic Frequency Control, AGC) is then activated and adjusts the production of the generators so as to compensate for the remaining frequency error and bring the tie-line power exchange back to the scheduled value. Specifically, the AGC output is distributed in a weighted way to certain generators. Hence, in the new steady state value, the power setpoint of these generators is changed by a certain percentage of the active power imbalance. In the current energy market, this percentage is the result of contracting agreements between producers and the TSO concerning the secondary frequency control reserves. The product of these weights with the worst case imbalance results in the amount of reserves that each generating unit should provide. In the sequel we will refer to the vector that includes these weights as the *distribution vector*. To encode the change of the generating output, we define the power correction term $R^i$,

$$R^i = \quad (2)$$
$$\begin{cases} d_{up} \max_+(-P^i_m) - d_{down} \max_+(P^i_m) & \text{if } i \in \mathcal{I}^l_{out} \cup \mathcal{I}^L_{out} \\ \tilde{d}^i_{up} \max_+(-P^i_m) - \tilde{d}^i_{down} \max_+(P^i_m) & \text{if } i \in \mathcal{I}^G_{out} \end{cases}$$

where $\max_+(\cdot) = \max(\cdot, 0)$. Variable $P^i_m \in \mathbb{R}$ denotes the generation-load mismatch, which for each outage is defined as

$$P^i_m = \begin{cases} P_w - P^f_w & \text{if } i \in \mathcal{I}^l_{out} \text{ or } i = 0 \\ P_w - P^f_w + P^i_L & \text{if } i \in \mathcal{I}^L_{out} \\ P_w - P^f_w - P^i_G & \text{if } i \in \mathcal{I}^G_{out} \end{cases}$$

For line outages, $P^i_m$ is just the deviation of wind power from the forecast $P^f_w$, whereas for load/generator outages the power of the lost component is also taken into account. Note that $P^i_L, P^i_G \in \mathbb{R}$ denote the element of $P_L \in \mathbb{R}^{N_L}$, $P_G \in \mathbb{R}^{N_G}$ which corresponds to the failed component $i$.

Vectors $d_{up}, \tilde{d}^i_{up} \in \mathbb{R}^{N_G}_+$, ($d_{down}, \tilde{d}^i_{down} \in \mathbb{R}^{N_G}_+$) represent the distribution vectors. The sum of their elements is one, and if a generator is not contributing to the AGC, the corresponding element in the vector is zero. The indices up and down are used to distinguish between the up and down spinning reserves. For every generator outage $i \in \mathcal{I}^G_{out}$ a different distribution vector is defined. The reason is that in such cases, a different component of $\tilde{d}^i_{up}, \tilde{d}^i_{down}$ turns to zero (the one corresponding to the outage), and the other elements are recalculated so that they sum to one.

Having defined the correction term $R^i$, the worst case up-down spinning reserves of each generating unit can be calculated in a similar way, but considering the worst case values of $P_m^i$. This can be done by taking the extreme values of the correction term among all contingencies, and using probabilistic bounds for the wind power error (see next section). For example, if we are given $\underline{y}, \overline{y} \in \mathbb{R}_+$ such that $-\underline{y} \leq P_w - P_w^f \leq \overline{y}$ (probabilistic bounds of this type will be computed in the next subsection), we have

$$R_{up} = \max\big(d_{up}\underline{y}, \max_{i\in\mathcal{I}_{out}^G} \tilde{d}_{up}^i(\underline{y} + P_G^i)\big), \tag{3}$$

$$R_{down} = \max\big(\max_{i\in\mathcal{I}_{out}^L} d_{down}(\overline{y} + P_L^i), \max_{i\in\mathcal{I}_{out}^G} \tilde{d}_{down}^i(\overline{y} - P_G^i)\big), \tag{4}$$

where the maximum operators in (3), (4) are assumed to apply elementwise. These expressions for the reserves are valid since it was assumed that all elements of the distribution vectors are non-negative (i.e. no network congestion).

### C. Probabilistic security constrained reserve scheduling

We consider an optimization horizon $N_t = 24$ with hourly steps, and introduce the subscript $t$ in our notation to characterize the value of the quantities defined in the previous section for a given time instance $t = 1, \ldots, N_t$. Let $C_1, C_2, C_{up}, C_{down} \in \mathbb{R}^{N_G}$ be generation and reserve cost vectors, and $[C_2]$ denote a diagonal matrix with vector $C_2$ on the diagonal. As also defined in [20], a quadratic form for the production cost is considered, whereas motivated by [17] the reserve cost was considered to be linear.

For each step $t$ of the optimization problem, define the vector of decision variables to be $x_t = \Big[P_{G,t}, d_{up,t}, d_{down,t}, [\tilde{d}_{up,t}^i]_{i\in\mathcal{I}_{out}^G}, [\tilde{d}_{down,t}^i]_{i\in\mathcal{I}_{out}^G}, \underline{y}_t,\ \overline{y}_t\Big]^T \in \mathbb{R}^{2N_G^2+3N_G+2}$. The resulting optimization problem is given by

$$\min_{x_t:t=1:N_t} \sum_{t=1}^{N_t} \Big(C_1^T P_{G,t} + P_{G,t}^T[C_2]P_{G,t} + C_{up}^T R_{up,t} + C_{down}^T R_{down,t}\Big), \tag{5}$$

subject to

1) Power balance constraints: For all $t = 1, \ldots, N_t$, $\mathbf{1}^T(C_G P_{G,t} + C_w P_{w,t}^f - C_L P_{L,t}) = 0$. Following the discussion of the previous subsection, this constraint encodes the fact that the power balance in the network should be always satisfied when $P_{w,t} = P_{w,t}^f$.

2) Generation limits: For all $t = 1, \ldots, N_t$, $P_{\min} \leq P_{G,t} \leq P_{\max}$, where $P_{\min}, P_{\max} \in \mathbb{R}^{N_G}$ denote the minimum and maximum generating capacity of each unit.

3) Distribution vector constraints: For all $t = 1, \ldots, N_t$

$$d_{up,t}, d_{down,t} \geq 0, \tag{6}$$

$$\tilde{d}_{up,t}^i, \tilde{d}_{down,t}^i \geq 0,\ \text{for all}\ i \in \mathcal{I}_{out}^G, \tag{7}$$

$$\mathbf{1}^T d_{up,t} = 1, \mathbf{1}^T d_{down,t} = 1, \tag{8}$$

$$\mathbf{1}^T \tilde{d}_{up,t}^i = 1, \mathbf{1}^T \tilde{d}_{down,t}^i = 1,\ \text{for all}\ i \in \mathcal{I}_{out}^G. \tag{9}$$

Recall that the element of $\tilde{d}_{up,t}^i, \tilde{d}_{down,t}^i$ corresponding to the tripped generator of the failure $i$ is set to zero. Note that, as discussed in Section II.B, finding a feasible solution in case of congestion may require removing constraints (6), (7).

4) Probabilistic constraints: For all $t = 1, \ldots, N_t$

$$\mathbb{P}\Big(P_{w,t} \in \mathbb{R} | -\overline{P}_{line}^i \leq B_f^i \begin{bmatrix} (\widetilde{B}_{BUS}^i)^{-1}\widetilde{P}_t^i \\ 0 \end{bmatrix} \leq \overline{P}_{line}^i, \quad P_{\min} \leq P_{G,t}^i + R_t^i \leq P_{\max}, \quad -\underline{y}_t \leq P_{w,t} - P_{w,t}^f \leq \overline{y}_t, \quad \text{for all}\ i = 0, \ldots, N_{out}\Big) \geq 1 - \epsilon, \tag{10}$$

where $\widetilde{P}_t^i$ is defined in (1), $R_t^i$ is given in Section II.B, and $\underline{y}_t, \overline{y}_t \geq 0$ for all $t = 1, \ldots, N_t$ are the probabilistic bounds of the wind power error i.e. difference between the wind power and its forecasted value.

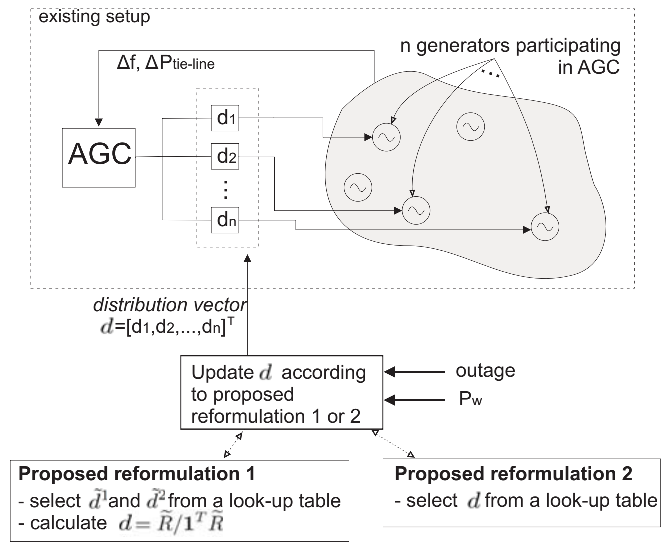


Fig. 1. Schematic diagram of the reserve scheduling algorithm.

The first constraint inside the probability is the standard transmission capacity constraints for each outage $i$. The second constraint provides guarantees that the scheduled generation dispatch plus the reserve contribution $R_t^i$ will not result in a new operating point outside the generation capacity limits. The last constraint of (10) enables us to compute probabilistic bounds on the maximum positive and negative wind deviation.

It should be noted that implicitly an additional functionality in the operation of the secondary frequency controller was assumed. Namely, in the case of a generator outage the operator of the system needs to monitor both the production of the tripped plant and the deviation of the wind power form its forecasted value, and using (2) as a look-up table, select the appropriate distribution vector, among those computed in the optimization problem (Fig. 1).

## III. A TRACTABLE PROBLEM FORMULATION

There are two main challenges when attempting to solve problem (5)-(10). The first arises from the presence of bilinear terms due to the products of $\tilde{d}_{up,t}^i, \tilde{d}_{down,t}^i$ and $P_{G,t}$ in (2) and (5). The second one arises from the presence of the chance constraint (10). To tackle the latter, the so called scenario approach will be employed in Section III.C. To alleviate the non-convexity introduced by the bilinearity two alternative convex reformulations will be introduced in Sections III.A and III.B.

### A. Proposed reformulation 1

Assume that in the case where $i \in \mathcal{I}_{out}^G$ we can distinguish between the mismatch that corresponds to wind deviation and the one which occurs due to a generator outage. For $i \in \mathcal{I}_{out}^G$, we would thus have

$$\begin{aligned}\widetilde{R}_t^i = \tilde{d}_{up,t}^{1,i} \ \max_{+}(P_{w,t}^f - P_{w,t}) \\ - \tilde{d}_{down,t}^{1,i} \ \max_{+}(P_{w,t} - P_{w,t}^f) + \tilde{d}_{up,t}^{2,i} P_{G,t}^i. \end{aligned} \quad (11)$$

No $\tilde{d}_{down,t}^{2,i}$ vector needs to be introduced, since $P_{G,t}^i$ is always positive. By reformulating (3)-(4) in a similar way, $\tilde{d}_{up,t}^{2,i} P_{G,t}^i$ becomes the only bilinear term, which appears both in the constraints and the objective function. Setting $z_t^i = \tilde{d}_{up,t}^{2,i} P_{G,t}^i \in \mathbb{R}^{N_G}$ as a new decision variable, and defining the new decision vector $\tilde{x}_t = [P_{G,t}, d_{up,t}, d_{down,t}, [\tilde{d}_{up,t}^{1,i}]_{i \in \mathcal{I}_{out}^G}, [\tilde{d}_{down,t}^{1,i}]_{i \in \mathcal{I}_{out}^G}, [z_t^i]_{i \in \mathcal{I}_{out}^G}, \underline{y}_t, \ \overline{y}_t]^T \in \mathbb{R}^{3N_G^2 + 3N_G + 2}$, makes the problem (5)-(10) linear in $z_t^i$, and hence convex (with a chance constraint). Once the solution to this problem is computed, $\tilde{d}_{up,t}^{2,i}$ is calculated as $\tilde{d}_{up,t}^{2,i} = z_t^i / P_G^i$ if $P_G^i$ is not equal to zero, and is zero otherwise.

For real time operation, the look-up table interpretation (discussed in Section III.C) may be adopted. Given then a mismatch $P_{m,t}^i = (P_{w,t} - P_{w,t}^f) - P_{G,t}^i$, the participation of each unit in compensating $P_{m,t}^i$ can be determined a posteriori by $\widetilde{R}_t^i / \mathbf{1}^T \widetilde{R}_t^i$. This requires knowledge of the mismatch terms.

Using this reformulation, a convex problem is achieved at the expense of a more conservative reserve schedule. This is due to the fact that $P_{w,t} - P_{w,t}^f, P_{G,t}^i$ are treated separately, leading to reserves of higher cost. To see this, consider the case where $P_{w,t} - P_{w,t}^f \geq 0$. The proposed formulation would lead to $|P_{w,t} - P_{w,t}^f| + |P_{G,t}^i|$ MW of reserves, whereas only $|P_{w,t} - P_{w,t}^f - P_{G,t}^i|$ MW are needed.

### B. Proposed reformulations 2

In this subsection we overcome the bilinearity problem by using an iterative algorithm (see Algorithm 1). We first attempt to identify a feasible solution of the problem, by starting from an arbitrarily chosen power schedule $P_{G,t}^0$ (stages are decoupled, so we focus on a specific time instance). At iteration $k$ of the algorithm, we fix $P_{G,t}^{k,i}$ *only* in (2) to the value obtained in the previous iteration. Solving then (5)-(10) a new solution $x_t^k$ is computed, and $P_{G,t}^k$ is updated accordingly. If the algorithm converges, its fixed point $x_t^{k^*}$ will be a feasible solution of the initial problem.

At a second step, we use an alternating iterative scheme to refine the resulting feasible solution in terms of cost. At iteration $k$ we first fix $\tilde{d}_{up,t}^{k,i}, \tilde{d}_{down,t}^{k,i}$ to the values obtained at the previous step of the algorithm, and obtain $\left[P_{G,t}^k, d_{up,t}^k, d_{down,t}^k, \underline{y}_t^k, \ \overline{y}_t^k\right]^T$ by solving (5)-(10). We then fix $P_{G,t}^k$ to the computed value in all equations it appears, and solve for $\left[d_{up,t}^k, d_{down,t}^k, [\tilde{d}_{up,t}^{k,i}]_{i \in \mathcal{I}_{out}^G}, [\tilde{d}_{down,t}^{k,i}]_{i \in \mathcal{I}_{out}^G}, \underline{y}_t^k, \ \overline{y}_t^k\right]^T$ instead. The entire process is then repeated until convergence. For a better understanding, Fig. 2 shows how the power dispatch of each unit, and the obtained objective value change per iteration, for the benchmark problem introduced in the next section. After 3 iterations the first part converges, whereas for the second one only one iteration is needed. As expected, the cost is decreasing monotonically in the second part.

**Algorithm 1**

```
——— Part 1 ———
Initialization.
Set P^0_{G,t} (e.g. P^0_{G,t} = 0),
k = 1.
Repeat until convergence
    Set P^{k,i}_{G,t} = P^{k-1,i}_{G,t}, ∀i ∈ I^G_out, only in (2),
    Compute x^k_t solving (5)-(10),
    Update P^k_{G,t},
    k = k + 1.
end
Return converged solution x^{k*}_t
——— Part 2 ———
Initialization.
Set d̃^{0,i}_{up,t} = d̃^{k*,i}_{up,t}, d̃^{0,i}_{down,t} = d̃^{k*,i}_{down,t}, ∀i ∈ I^G_out,
k = 1.
Repeat until convergence
    Set d̃^{k,i}_{up,t} = d̃^{k-1,i}_{up,t}, d̃^{k,i}_{down,t} = d̃^{k-1,i}_{down,t}, ∀i ∈ I^G_out, in (2),
    Compute [P^k_{G,t}, d^k_{up,t}, d^k_{down,t}, y_t^k (lower), ȳ^k_t]^T
    solving (5)-(10),
    Fix P^k_{G,t} in (5)-(10),
    Solving (5)-(10), compute
    [d^k_{up,t}, d^k_{down,t}, [d̃^{k,i}_{up,t}]_{i∈I^G_out}, [d̃^{k,i}_{down,t}]_{i∈I^G_out}, y_t^k (lower), ȳ^k_t]^T
    k = k + 1.
end
```

Note that the first part of Algorithm 1 is a heuristic scheme applied to identify a feasible solution, and no convergence guarantees can be provided. The second part of the algorithm converges monotonically, since it is a bilinear descent iteration; the limit point however is not guaranteed to be the global optimum of the original bilinear problem. Note that every optimization involved in Algorithm 1 is now convex.

### C. The scenario approach

Using either of the previous reformulations, a convex chance constrained optimization program is constructed. To obtain a solution for this problem, we use the so called scenario approach [24]. The scenario approach replaces the chance constraint with a finite number of hard constraints, while offering probabilistic performance guarantees. The resulting problem is convex and can be then solved efficiently using existing numerical tools [25]. The authors of [21]

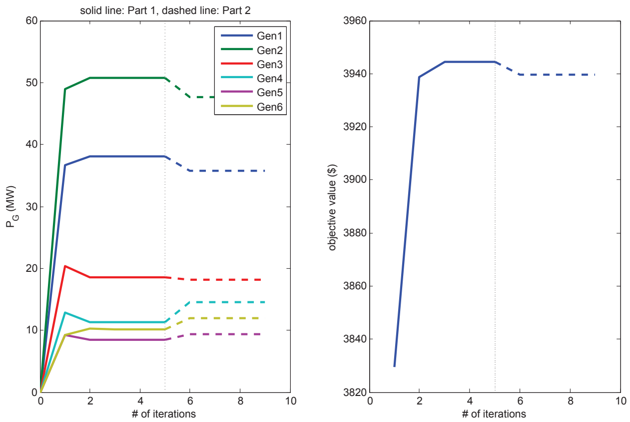


Fig. 2. Illustration of Algorithm 1 for one hour of the simulated data, initialized with $P_{G,t}^{0} = 0$. For the first part, the power dispatch of each unit and the obtained objective value converge after 3 iterations, whereas for the second one only 1 iteration is needed.

provide a bound on the number of scenarios one needs to generate to maintain these $\epsilon$-type guarantees. That is

$$N_s \geq \frac{2}{\epsilon}\Big(\ln\frac{1}{\beta} + N_d\Big), \tag{12}$$

where $\epsilon \in (0, 1)$ is the violation parameter determining the desired probability level (see (10)). Variable $N_d$ denotes the number of decision variables, which for this case is equal to the dimension of $x_t$ (similarly for $\tilde{x}_t$). Parameter $\beta$ characterizes the confidence with which, the solution of the problem will violate the chance constraint with probability at most $\epsilon$.

The number of scenarios $N_s$ grows with the number of decision variables. Following [26], for a given $\epsilon$ and $\beta$, we could first compute probabilistic bounds for $P_{w,t} - P_{w,t}^{f}$, and then use them to compute the solution of the robust counterpart of (5)-(10). That way, we only need to use $N_s$ with $N_d = 2$, since we have only two decision variables. This modification leads to an equivalent problem, in the sense that that the same probabilistic guarantees are achieved, but by generating less scenarios.

## IV. Simulation Results

In this section we evaluate the performance of our approach by applying it to the IEEE 30-bus network [22]. The benchmark includes $N_b = 30$ buses, $N_G = 6$ generators, $N_l = 41$ lines, and is modified to include a wind power generator connected to bus 22. To generate the required wind power scenarios we employed a Markov chain-based model, as in [27], [19], whereas to compute the solution of the problem numerically, the solver CPLEX [25] was used via the MATLAB interface YALMIP [28].

We first compare the two alternative reformulations in terms of cost. Fig. 3 shows the cost of the production and the cost of the reserves for one day of the simulated data. It should be remarked that for both methods, the total cost (solid line) is determined mainly by the cost of the generation schedule (dashed line), whereas their difference indicates the

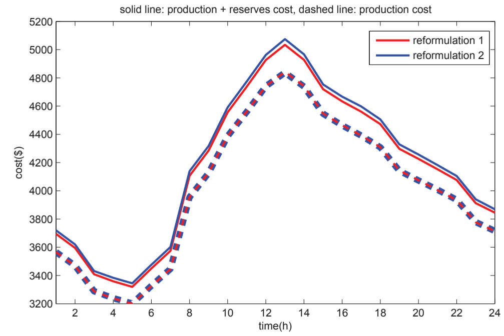


Fig. 3. Total cost (solid line) and cost of the generation schedule (dashed line); the difference indicates the cost of the reserves. The "red" curves correspond to reformulation 1, which leads to slightly lower total cost compared to reformulation 2 ("blue"). For both reformulations the production cost is the same, hence the dashed lines overlap. For the scenario approach we used $\epsilon = 0.1$ and $\beta = 10^{-4}$.

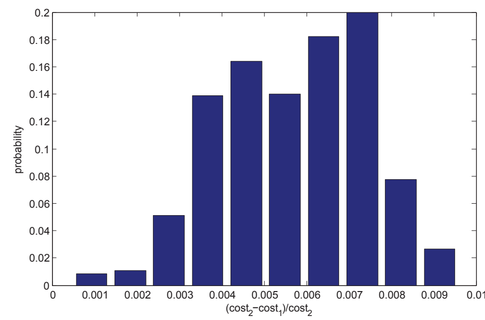


Fig. 4. Distribution of the percentage of cost improvement, when using reformulation 1 against reformulation 2 for 30 days of hourly measured data.

cost of the reserves. For both approaches the production cost is the same, hence the dashed lines overlap. As expected, the total costs follows closely the load profile that was used throughout the simulations. Despite its conservative nature, the proposed reformulation 1 ("red") leads to slightly lower total cost compared to reformulation 2 ("blue"). This is explained by the fact that in the first case the global optimum is identified, whereas the iterative procedure of Algorithm 1 might get stuck in local optima. Fig. 4 depicts the distribution of the percentage of cost improvement, using reformulation 1 against reformulation 2 for 30 days of hourly measured data. These correspond to different forecast and actual wind power data, which were extracted from a normalized wind power data-set corresponding to the total wind power in-feed of Germany over the years $2007 - 2008$. This is different from the data used to train the Markov chain-based model of the wind power error dynamics, which was used to generate scenarios.

Irrespective of the reformulation, the amount of daily load shed ("red") and wind generation spilled ("blue") is shown in Fig. 5, as this was computed using the actual wind power data (not included in the scenarios used for optimization purposes). The empirical probability of load shedding or wind generation spillage, calculated as the number of hours

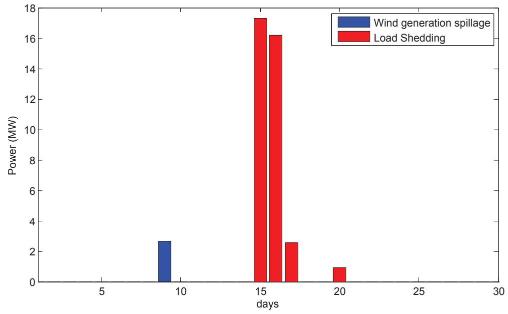


Fig. 5. Load shedding ("red") and wind generation spillage ("blue") per day, for 30 days of the simulated data.

that these actions had to be executed over the total number of simulated hours ($30 \times 24$), was found to be $3.75\%$. Apart from the cases of load shedding and wind generation spillage no other insecure instance was encountered.

## V. Concluding remarks

In this paper a new methodology for solving security constrained reserve scheduling problems for systems with fluctuating generation, is proposed. Two different convex reformulations are derived, and the resulting chance constrained optimization program is solved using the scenario approach. The proposed reformulations were evaluated via Monte Carlo simulations on the IEEE 30-bus network.

Current work concentrates toward identifying a more suitable convex reformulation of the problem, and studying the convergence properties of the iterative, heuristic alternative. Moreover, we aim to substitute the underlying DC power flow set-up with a convex AC optimal power flow model, and investigate the potential of decentralizing the developed algorithm.

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