Asymmetries in Business Cycles and the Role of Oil Prices

Betty C. Daniel[*,1], Christian M. Hafner[2], Hans Manner[3], and Léopold Simar[2]

[1]*Department of Economics, The University at Albany*
[2]*Institute of Statistics and CORE, Université catholique de Louvain*
[3]*Department of Social and Economic Statistics, University of Cologne*

October 16, 2012

Abstract

We study business cycle asymmetries in a sample of eleven OECD countries by allowing total factor productivity to be composed of a symmetric innovation, as in Dynamic Stochastic General Equilibrium Models, and a negative asymmetric one, as first proposed by Barro (2006) to model rare disasters. We adapt Stochastic Frontier Analysis from its standard micro applications to estimate whether or not innovations have negative asymmetries. Likelihood ratio statistics and variance ratios imply that all countries with net energy imports have significant negative asymmetries, while other countries do not. We present a theoretical model in which capacity utilization can interact with concavity in production to produce asymmetries for large oil price increases, but not for small ones. We find that conditioning on Hamilton's (2011) net oil price increase variable reduces, but does not eliminate, evidence of negative asymmetries for net energy importers. Additional conditioning on financial crises, as suggested by Barro (2006), completely eliminates evidence of negative asymmetry.

Keywords: Solow residuals, stochastic frontier analysis, oil price shocks, financial crises
JEL Classification: E32, C22, C13

---

[*]Corresponding author, b.daniel@albany.edu

1 Introduction

Are business cycles asymmetric with expansions fundamentally different from recessions? This question is central to understanding business cycles and has received considerable attention empirically and theoretically.

Empirical work has taken two approaches to determining asymmetry, one measuring amplitude and frequency of expansions and recessions, with the other postulating Markov switching between expansion and recession regimes. NBER business cycle dating generates recessions with smaller duration and frequency than booms for the US. McKay and Reis (2008) confirm that contractions are briefer and more violent when measured by employment but not when measured by GDP. Markov switching models (Hamilton 1989, Raymond and Rich 1997, Clements and Krolzig 2002, Engemann et al. 2011) similarly find that recession regimes are systematically different from expansion regimes.

Dynamic Stochastic General Equilibrium (DSGE) models were explicitly developed to study business cycles. These models view the business cycle as the response of capital and labor, and, therefore, of output, to exogenous symmetric productivity shocks. In these models symmetric shocks to total factor productivity (TFP), measured as Solow residuals in the production function, are the major determinants of business cycles. The standard DSGE model is linearized, and, therefore, has no role for asymmetries.

In an attempt to explain the equity premium puzzle created by DSGE modeling, Barro (2006) postulates an asymmetric time series process for total productivity. Specifically, he assumes that the log of total factor productivity evolves as a first order autoregressive process with a constant mean and an error which has two components. One component is i.i.d normal, capturing symmetric productivity shocks, as in the standard DSGE model. The second component has a distribution containing only negative values and captures rare disasters. Barro defines disasters to include war, natural disasters, financial crises – any event severe enough to create at least a fifteen percent cumulative fall in GDP.

In this paper, we use an asymmetric specification of total factor productivity to propose a third empirical approach to measuring business cycle asymmetries. As in Barro (2006), we assume that total factor productivity has a symmetric i.i.d. component and an asymmetric component with only negative values. In contrast to Barro, we do not require the asymmetric negative component of the error to be large, only that its distribution be restricted to negative values. The one-sided error term introduces the possibility that some recessions are generated by a different stochastic process than the one generating booms and (other) recessions.

We introduce stochastic frontier analysis, borrowed from the micro literature on productivity analysis, to decompose innovations in differenced log Solow residuals into a symmetric and an asymmetric component. This technique assumes that the innovation is a composite of a one-sided negative error and a normal two-sided error, as in Barro (2006). The standard use for the technique is to measure inefficiency in a cross-section, where inefficiency is viewed as realizations of the one-sided error. We use the technique of separating the error into two components to measure asymmetry in a time series, not inefficiency in a cross section. If Solow residuals have an asymmetric component, then we should attribute some of the variance to the one-sided error.

To apply the stochastic frontier methodology to aggregate time series data, we amend the micro technique to deal with the persistence in macro data. We first difference the logarithm of the Solow residual to generate a stationary time series, and we modify the standard stochastic frontier model to allow for autocorrelation. Our sample consists of eleven OECD countries which had quarterly data available back at least to the early 1980's on GDP, employment, and investment. We compare the restricted model without asymmetries to the unrestricted model using a likelihood ratio test. Additionally, we estimate the ratio of variances for the two components of the innovations to compare the extent of asymmetries across countries.

Next, we use the variation in asymmetries across countries to understand the underlying causes of the asymmetry. We find that all countries which are net importers of energy have negative asymmetries and that no countries which are net exporters do. This suggests that energy-importing countries are structurally different from energy-exporting ones. There is a large empirical literature on output responses to oil prices. Hamilton (1983) demonstrated that all but one post World War II recession in the US were preceded by a large rise in oil prices. Mork (1989) and Hamilton (2011) have argued that business cycles exhibit asymmetric responses to oil price shocks, but Kilian and Vigfusson (2011a,b) dispute this claim. Kilian (2008) finds negative responses in the US to oil events, measured as international supply disruption. Engemann et al. (2011) argue that the probability of switching to a recession regime in a Markov switching model depends on the price of oil for the US and several other OECD countries.

The mixed evidence on the empirical importance of oil prices in affecting business cycles is accompanied by strong criticism at the theoretical level of the hypothesis that oil prices have any effect. Although Hamilton (2011) offers theoretical explanations for asymmetric effects of oil prices on GDP, Kehoe and Ruhl (2008) and Barsky and Kilian

(2004) argue that theory implies no effects of oil prices on Solow residuals. We present a theoretical model to show that large oil price shocks can interact with changes in capacity utilization to create negative asymmetry in the Solow residual for a country with net oil imports. The model yields asymmetries only for large shocks and relies on the interaction of endogenous capacity utilization and concavity in the production function.

Next, we determine whether non-linear conditioning on oil prices, using Hamilton's (2011) net oil price increase variable, can eliminate the negative asymmetries for energy importing countries. We find that this conditioning reduces negative asymmetries, measured by variance ratios, for all energy-importing countries except Korea, and completely eliminates negative asymmetries for many. This provides evidence in favor of the hypothesis that one cause of business cycle asymmetries is large changes in oil prices for energy importers. However, negative asymmetries do remain for some countries.

Finally, we consider other causes for negative asymmetry by conditioning on financial crises, following Barro (2006). Conditioning only on financial crises, and not on oil prices, reduces negative asymmetries, but does not eliminate them. When we condition on financial crises and on large oil price increases, all evidence of negative asymmetries vanishes.

The rest of the paper is organized as follows. In Section 2 we introduce our econometric methodology. Section 3 presents the data and the baseline empirical results. In Section 4, we develop a theoretical model which yields a role for asymmetric responses to large changes in oil prices for net energy importers. Section 5 presents additional results on asymmetries after conditioning on oil prices and on financial crises. Section 6 concludes.

2 Econometric Methodology

In this section, we present an econometric model to investigate and test whether Solow residuals have an asymmetric component. Asymmetry is a key issue in the literature on production efficiency. The classical model of stochastic frontier analysis (SFA), proposed simultaneously by Meeusen and van den Broek (1977) and Aigner et al. (1977) to estimate inefficiencies across individual firms, incorporates a composite error term which, under the null hypothesis of efficiency, is symmetric. We modify this model framework to include additional dynamic effects that take into account potential serial correlation of Solow residuals and use it to estimate the ratio of variances of the symmetric and asymmetric error components.

2.1 Model specification

We follow Barro (2006) and assume that the first difference of logged productivity is determined by a composite error. We use data described below to construct a measure of the Solow residual (SR_{Mt}), and express the log-differenced Solow residual (q_t) as

$$q_t = \log(SR_{Mt}) - \log(SR_{Mt-1}) = \mu_t + w_t, \tag{1}$$

where, conditional on the information set generated by lagged Solow residuals, μ_t is the conditional mean of q_t, and w_t is a composite error term with mean zero. We assume that this error term can be decomposed as

$$w_t = v_t - u_t, \tag{2}$$

where v_t is a Gaussian random variable, $v_t \sim N(\mu_u, \sigma_v^2)$ with mean $\mu_u > 0$. The second term u_t is an exponentially distributed (positive) random variable with mean μ_u. Note that by construction the mean of w_t is restricted to be zero. Other typical choices for the distribution of the one-sided error term u_t are half-normal, truncated normal, or gamma (Kumbhakar and Lovell 2000, Murillo-Zamorano 2004). For our data, described in Section 3, we found that the exponential distribution provides the best fit, so we restrict attention to this model. The presence of the term u_t implies that the distribution of the composite error w_t is asymmetric, except for the degenerate case in which $\mu_u = 0$ and the distribution reduces to a normal with variance σ_v^2.

The specification of the conditional mean μ_t depends on potential serial correlation of the growth rates of Solow residuals q_t. We consider autoregressive models of order p, AR(p), given by

$$q_t = \beta_0 + \sum_{i=1}^{p} \beta_i q_{t-i} + w_t, \tag{3}$$

where the usual stationarity conditions are assumed to be satisfied, and w_t is the composite error term defined by equation (2).[1]

2.2 Estimation and evaluation

Estimation of model (3), including the autoregressive coefficients and parameters in the distributionals for u_t and v_t can be conveniently done by maximum likelihood.

---

[1]In our application we experimented with MA terms and found them unnecessary to capture dynamics.

A closed form expression for the density of the composite error $w_t = v_t - u_t$ exists and is given by (see Kumbhakar and Lovell 2000)

$$f(w_t; \mu_u, \sigma_v) = \frac{1}{\mu_u} \Phi\left(-\frac{w_t - \mu_u}{\sigma_v} - \frac{\sigma_v}{\mu_u}\right) \exp\left(\frac{w_t - \mu_u}{\mu_u} + \frac{\sigma_v^2}{2\mu_u^2}\right), \tag{4}$$

where Φ denotes the standard normal distribution function and μ_u is the parameter of the exponential distribution, which has mean equal to μ_u and variance equal to μ_u^2. The mean of u_t is subtracted from w_t to account for the fact that, unlike in the classical stochastic frontier model, w_t is assumed to have mean zero. The log-likelihood function of the joint model, including the autoregressive dynamics, can be obtained in a straightforward way.

A primary goal of this paper is to test whether the model, allowing for a one-sided error component in the AR innovations, outperforms the standard model with symmetric errors. The model with symmetric errors is nested in the model above when the parameter μ_u equals zero. Thus, one can test the null hypothesis that the benchmark model performs as well as our model by doing a likelihood ratio test. With the restriction $\mu_u \geq 0$, however, the parameter to test is on the boundary of the parameter space under the null hypothesis. Lee (1993) showed that, in this case, the likelihood ratio statistic asymptotically follows a mixture of a χ^2 distribution with one degree of freedom and a point mass of $1/2$ at zero.

Estimation of this model is problematic when the sample skewness is positive. Aigner et al. (1977) demonstrated that theoretically in such situations the MLE of μ_u will converge to zero, and Lee (1993) showed that in this case the information matrix is singular, which implies that maximum likelihood standard errors cannot be calculated. In practice, when the residuals have positive skewness, the MLE using (4) will either fail to converge or will converge to a local maximum. For cases in which the sample skewness of the residuals of the autoregressive model with Gaussian innovations is positive, we extend the model to explicitly allow for positive skewness. We allow $-u_t$ to follow an exponential distribution, such that u_t has a negative mean μ_u and standard deviation $|\mu_u|$. This leads to the composite density of w_t,

$$\begin{aligned} f(w_t; \mu_u, \sigma_v) &= \frac{1}{\mu_u} \Phi\left(-\frac{w_t - \mu_u}{\sigma_v} - \frac{\sigma_v}{\mu_u}\right) \exp\left(\frac{w_t - \mu_u}{\mu_u} + \frac{\sigma_v^2}{2\mu_u^2}\right) I(\mu_u \geq 0) \\ &\quad - \frac{1}{\mu_u} \Phi\left(\frac{w_t - \mu_u}{\sigma_v} + \frac{\sigma_v}{\mu_u}\right) \exp\left(\frac{w_t - \mu_u}{\mu_u} + \frac{\sigma_v^2}{2\mu_u^2}\right) I(\mu_u < 0), \end{aligned} \tag{5}$$

which for $\mu_u > 0$ corresponds to the one given in (4) with negative skewness, and for $\mu_u < 0$ is the mirrored version with positive skewness. $I(\cdot)$ denotes the indicator function. To illustrate the shape of these densities, some examples are depicted in Figure 1. Substituting (5) for equation (4) avoids convergence problems of the estimation algorithm due

6

Figure 1: Density function (5) with $\sigma_v = 1$. Solid curve: $\mu_u = 0$ (Gaussian), long dashed curve: $\mu_u = 1.5$ (negative skewness), short dashed curve: $\mu_u = -1.5$ (positive skewness)
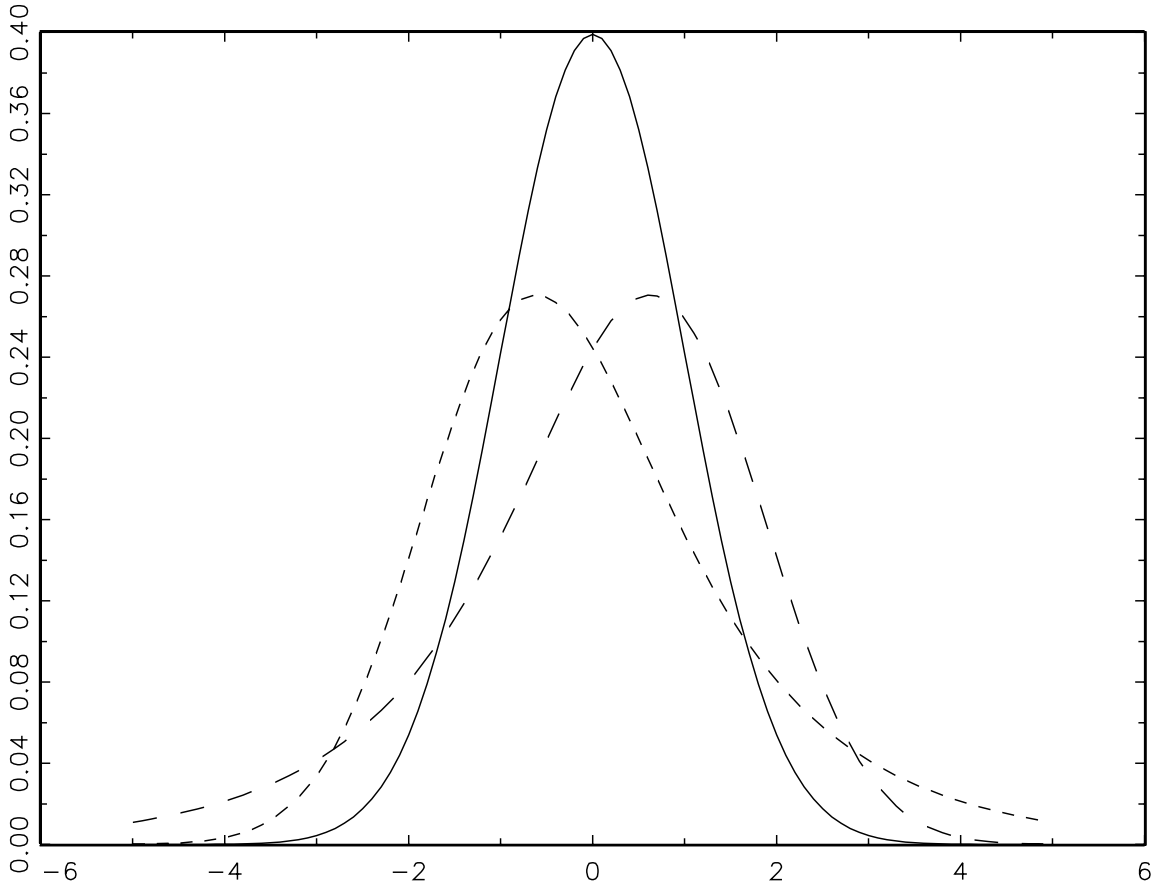
to positive sample skewness. Moreover, it is possible to use a standard χ^2 distribution for a likelihood ratio test because the parameter μ_u is no longer on the boundary under the null hypothesis. Even if the estimate of μ_u is negative, it could be insignificant and support the evidence of a symmetric error distribution. Note that this model specification is a vehicle to test our null hypothesis of interest, rather than our model of interest, which remains (4). We use equation (5) only when the estimation of equation (4) fails to converge due to positive skewness.

The relative importance of u_t can be measured by computing the variance ratio (VR) of the asymmetric component relative to the symmetric component as

$$VR = \frac{\mu_u^2}{\sigma_v^2},\tag{6}$$

7

which is estimated using the parameter estimates for the two error components. Note that the variance ratio measures the degree of asymmetry in the data with an increase in the ratio denoting more asymmetry. This is in contrast to the traditional stochastic frontier literature where the inefficiency of a firm is measured by *technical efficiency* (TE) defined by Battese and Coelli (1988) as $E[\exp(-u)|w]$. TE measures the distance from the efficient frontier or actual output divided by optimal efficient output. However, in our context TE cannot be interpreted, because the expectation of u is absorbed by the mean of v and consequently only the variance of u relative to the variance of v is relevant.

3   Empirical Analysis

The data set consists of seasonally-adjusted quarterly observations on constant-price GDP, investment, and employment for all OECD countries which had data dating back at least to the early 1980's. We include the following countries: Australia, Canada, France, Germany, Italy, Japan, Korea, Norway, Switzerland, United Kingdom and United States. OECD does not have data on labor hours, a preferable measure for the labor input, or on capital stock. The sample begins in 1973:Q1, with the exception of Korea (1983:Q1) and Switzerland (1976:Q1), and ends in 2011:Q3. Kilian and Vigfusson (2011b) argue that due to the regulation of nominal oil prices prior to 1973, the dynamic properties of oil prices in that period contain no information about the period following 1973, so that this seems a suitable starting point for our sample. The data set does not constitute a panel because units of measurement for each country's output and investment differ since they are measured in country-specific units of 2005 GDP. Data from the Penn World Tables does adjust cross-country data to comparable units, using purchasing power parity measures of relative prices, but that data exists only at annual frequency. Since we are interested in business cycle properties, quarterly frequencies are essential. Therefore, we estimate eleven separate equations, decomposing the innovations of each country's Solow residual into a symmetric and an asymmetric component.

To construct Solow residuals, we first construct measures of the capital stock. We use the perpetual inventory method, letting the initial value of the capital stock (K_0) be the steady-state equilibrium value with the growth rate (g) equal to the average of growth over the first ten years of the sample, annual depreciation (δ) at 0.07, following Easterly and Levine (2001), and initial investment equal to its initial value (I_0).[2] Subsequent values

---

[2]The initial capital stock becomes $K_0 = \frac{I_0}{g+\delta}$. Since the sample begins with 1973Q1, I_0 is investment

for capital are computed using the equation for the adjustment of the capital stock,

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{7}$$

To compute Solow residuals, we use employment as the measure of labor input and set capital's share at 0.35, following Stock and Watson (1999).

Augmented Dickey-Fuller tests with and without a trend do not reject the hypothesis of a unit root in Solow residuals for all countries. Therefore, we compute the first differences of the logarithm of the series. Before estimating our models we standardize the difference of log Solow residuals to have mean zero and variance equal to one. This standardization does not affect the estimation of the quantities of interest such as the skewness or the variance ratio. Empirical autocorrelations and partial autocorrelations (not reported) suggest that low order autoregressive models capture the dynamics in the data and that no significant autocorrelation is present for many series. The lag length was chosen using a combination of information criteria and the Ljung-Box test for residual autocorrelation up to 24 lags. We allowed for the possibility of dropping insignificant intermediate autoregressive lags to reduce the number of parameters to be estimated. However, we checked the robustness of our results with respect to that choice.

We present the estimation results from our baseline dynamic stochastic frontier (DSF) model defined in equations (1) to (3) in Table 1. The table contains the estimated parameters of the composite error term, the sample skewness of the residuals w_t, the log-likelihood of both the model restricted to symmetry (LL sym) and our dynamic stochastic frontier model (LL DSF), the likelihood ratio statistic (LR stat) for the null of symmetry along with its p-value, and the variance ratio (VR) implied by the estimated model. When the residuals from the Gaussian model exhibit positive skewness, we estimate the extended model (5), allowing computation of standard errors and construction of the likelihood ratios. Note, however, that in these cases rejection of the null hypothesis of symmetry provides evidence of positive asymmetry. Also recall that whenever we use the extended model (5) we use the χ^2 distribution to compute p-values rather than the mixture distribution with point mass at 0 that is required when relying on (4).

The likelihood ratio test rejects the null hypothesis of symmetry against negative asymmetry for all countries except Australia, Canada and Norway. For Australia the symmetric model is rejected, but in favor of the model with positive skewness. Canada and Norway have no asymmetries, while the remaining countries all have significant negative asymmetries.

---

in this first period.

9

It is interesting to consider which countries fail to exhibit negative asymmetries. Both Canada and Norway are oil exporters. Australia is a net oil importer, but a net exporter of energy. The UK's experience is mixed, with net exports from 1981 to 1988 and from 1993 to 2003, and net imports in other periods (Bolton 2010).

Therefore, the estimation provides evidence that all countries with some history of net energy imports have significant negative asymmetries. Countries which were consistently net exporters of energy do not. These results imply that oil prices could have a role in creating asymmetries in Solow residuals for countries which are net importers of energy.

This empirical finding poses a theoretical challenge. Barsky and Kilian (2004) argue that there is no theoretical justification for asymmetric effects of oil prices. Kehoe and Ruhl (2008) argue that although terms of trade and Solow residuals are strongly correlated, there is no theory linking oil prices to the Solow Residual. Below, we show that variable capacity utilization, combined with concavity in production, can provide the missing link.

Table 1: Estimation results of the DSF model

	Aus	Can	Fra	Ger	Ita	Jap	Kor	Nor	Swi	UK	US
μ_u	-0.57***	-0.37	0.50***	0.57***	0.59***	0.69***	0.56***	-0.56***	0.60***	0.64***	0.60***
σ_v	0.80***	0.85***	0.82***	0.80***	0.69***	0.71***	0.78***	0.80***	0.61***	0.74***	0.79***
skewness	0.318	0.116	-0.161	-0.519	-0.635	-0.781	-0.689	0.269	-2.631	-0.391	-0.262
LL sym.	-214.4	-202.5	-211.5	-216.6	-203.6	-218.0	-161.3	-212.8	-191.9	-218.0	-218.0
LL DSF	-212.5	-202.3	-210.0	-213.0	-199.1	-211.6	-156.0	-211.8	-172.6	-212.7	-215.9
LR stat.	3.838	0.360	3.020	7.265	9.010	12.73	10.42	1.918	38.52	10.56	4.302
p-val.	0.050	0.549	0.041	0.004	0.001	0.000	0.001	0.166	0.000	0.001	0.019
VR	0.513	0.196	0.376	0.501	0.740	0.949	0.518	0.495	0.963	0.750	0.581

Note: Table 1 reports the estimation results of model (4) defined in Section 2.1 consisting of an AR model with an error term composed of a one-sided exponentially distributed r.v. with parameter μ_u and a two-sided Gaussian r.v. with parameter σ_v. The data are growth rates of quarterly Solow residuals calculated as explained in Section 3. Skewness refers to the sample skewness of the residuals from the model with asymmetry, LL sym and LL DSF are the log-likelihood values of the model restricted to symmetry and the general model respectively, LR stat is the log likelihood ratio statistics for the null hypothesis of a symmetrically distributed error term, and VR refers to the ratio of the estimated variances of the one-sided and the two sided innovations. When sample skewness is positive, estimation of model (4) fails, and we present results from model (5). ***, ** and * refer to significance at the 1%, 5% and 10% confidence level, respectively.

4 A Theory for Asymmetric Effects of Energy Prices

We assume that all countries have some ability to produce their own energy, and that this ability varies across countries.The representative firm in each country chooses how much energy to produce based on the price of imported energy and its own production costs, and imports the remainder. We assume that the price of imported energy is exogenous to the representative firm.

Since imported energy's share is empirically rising in price, we follow Hassler et al. (2012) and specify a CES production function for output as

$$Y_t = \left[(1 - \gamma) \left[A_t K_t^\alpha L_t^{1-\alpha} \right]^{\frac{\epsilon-1}{\epsilon}} + \gamma \left[A_t^E E_t \right]^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{8}$$

where γ is imported energy's share in the event that $\epsilon = 1$, E_t is imported energy, A_t^E is technology in energy, and ϵ is the elasticity of substitution between the capital-labor composite and energy. We assume $\epsilon < 1$ to be consistent with empirical evidence that energy's share is increasing in price. A country which produces all of its own energy domestically will have $\gamma = 0$. We think about γ as a structural decision made by the representative firm based on its own production technology for energy and the imported price. This decision cannot be changed in the short run. However, if the price of energy gets too high, even with the current value for γ, the firm could be better off producing all of its own energy and could choose to do so. This puts an upper bound on the price for which the firm will import.

We assume that the energy price is low enough that the representative firm chooses to import. In this case, the firm chooses the quantity of imported energy input to maximize

$$\left[(1 - \gamma) \left[A_t K_t^\alpha L_t^{1-\alpha} \right]^{\frac{\epsilon-1}{\epsilon}} + \gamma \left[A_t^E E_t \right]^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} - p_t E_t,$$

where p_t is the relative price of energy in terms of production. The value for γ determines the curvature of production with respect to imported energy. First order conditions require that the marginal product of energy in production equal its relative price

$$\frac{\partial Y_t}{\partial E_t} = p_t,$$

yielding an expression for optimal energy as

$$E_t = Y_t p_t^{-\epsilon} \gamma^\epsilon \left(A_t^E \right)^{\epsilon-1}. \tag{9}$$

Using equation (9), energy's share can be expressed as

$$\frac{p_t E_t}{Y_t} = \gamma \left(\frac{p_t}{\gamma A_t^E} \right)^{1-\epsilon}. \tag{10}$$

Therefore, with $\epsilon < 1$, energy's share of production depends positively on its relative price.

Substituting equation (9) for E_t into equation (8) for production yields an expression for production as a function of the relative price of imported energy.

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} (1-\gamma)^{\frac{\epsilon}{\epsilon-1}} \left[1 - \gamma \left(\frac{p_t}{\gamma A_t^E} \right)^{1-\epsilon} \right]^{\frac{\epsilon}{1-\epsilon}} \tag{11}$$

Using equations (9) and (11), we can solve for energy demand as a function of price and the capital-labor composite as

$$E_t = (1-\gamma)^{\frac{\epsilon}{\epsilon-1}} \frac{A_t}{A_t^E} K_t^\alpha L_t^{1-\alpha} \left[\left(\frac{p_t}{\gamma A_t^E} \right)^{\epsilon-1} - \gamma \right]^{\frac{\epsilon}{1-\epsilon}}. \tag{12}$$

The firm will choose to import positive quantities of energy only if the term in brackets is positive, implying an upper bound on the energy price for which it chooses to import.

Derivatives of equations (11) and (12) illustrate that an increase in the price of imported energy reduces energy imports, thereby reducing production for a given value for capital and labor.

Kehoe and Ruhl (2008) explain that we must calculate the Solow Residual from value added, not from production. Value added, not total production, is the GDP concept. Value added (GDP) is given by production minus the value of imports in terms of production. Using equations (10) and (11) yields

$$GDP_t = Y_t - p_t E_t = Y_t \left(1 - \gamma \left(\frac{p_t}{\gamma A_t^E} \right)^{1-\epsilon} \right) = A_t K_t^\alpha L_t^{1-\alpha} (1-\gamma)^{\frac{\epsilon}{\epsilon-1}} \left[1 - \gamma \left(\frac{p_t}{\gamma A_t^E} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.$$

The Solow Residual is calculated as value added divided by the capital-labor input, yielding

$$SR_t = \frac{Y_t - p_t E_t}{K_t^\alpha L_t^{1-\alpha}} = A_t (1-\gamma)^{\frac{\epsilon}{\epsilon-1}} \left[1 - \gamma \left(\frac{p_t}{\gamma A_t^E} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}. \tag{13}$$

Taking the derivative of the Solow Residual in equation (13) with respect to the energy price yields

$$\frac{\partial SR}{\partial p_t} = -(1-\gamma)^{\frac{\epsilon}{\epsilon-1}} \frac{A_t}{A_t^E} \left[\left(\frac{p_t}{\gamma A_t^E} \right)^{\epsilon-1} - \gamma \right]^{\frac{\epsilon}{1-\epsilon}} = \frac{-E_t}{K_t^\alpha L_t^{1-\alpha}},$$

where the last equality uses equation (12) for optimal energy. Therefore, an increase in the price of imported energy reduces the Solow residual, reducing output for given capital and labor.

The second derivative of the Solow residual with respect to price is positive for $\epsilon < 1$, implying that the Solow residual is falling at an increasing rate in price. Since the Solow residual is continuous in price, there are no asymmetries for small changes in price. However, because the Solow residual is falling at an increasing rate in price, large increases in price will have asymmetrically large effects on the Solow residual compared with large decreases. The larger the oil price change, the more economically significant are the asymmetries.

We can understand the effect of energy price increases and decreases using a diagram. Figure 2 graphs production, equation (8), as a function of imported energy for a given value of the capital labor composite $\left(K_t^\alpha L_t^{1-\alpha}\right)$ as Y, and the cost of imported energy for a given price, P_j, as $P_j E$. Under the assumption that energy price is low enough for the firm to prefer imports, the profit maximizing use of energy occurs at E_0 where the slope of the production function equals the price of energy, equivalently, where the slope of Y equals the slope of $P_0 E$. The value for GDP_0 is given by the vertical distance between Y and PE at the point where the slopes are equal. The Solow residual is the value of GDP_0 divided by the capital-labor composite $\left(K_t^\alpha L_t^{1-\alpha}\right)$.

Now, consider the effect of an increase in the price of imported energy (P_j) on GDP, holding the capital-labor composite constant, under the assumption that the price increase is small enough that the representative firm continues to import energy. As price increases to $P_1 > P_0$, the slope of $P_j E$ rises, and equilibrium occurs where the slope of the production function is higher to match the higher energy price, at E_1. Therefore, holding the capital-labor composite constant, energy use falls and GDP falls from GDP_0 to GDP_1. Since GDP is lower for a given value for $K_t^\alpha L_t^{1-\alpha}$, the Solow residual is lower. A decrease in price would reduce the slope of the $P_j E$ line, increasing GDP, but by a smaller amount than the same size price increase raises GDP, due to concavity of the production function.

However, Kehoe and Ruhl (2008) explain that this measure of the Solow Residual, based on the relative price of imported energy in terms of production, is not the measure in the national income accounts. The official measure of real value added is not in terms of a numeraire (production in our example) but in terms of constant dollars. Letting $\bar{p}$ denote the dollar price of imported energy in the base year with the dollar price of output at unity, and using equation (9), we can express value added in constant dollars, which

Figure 2: Real GDP and Measured Real GDP
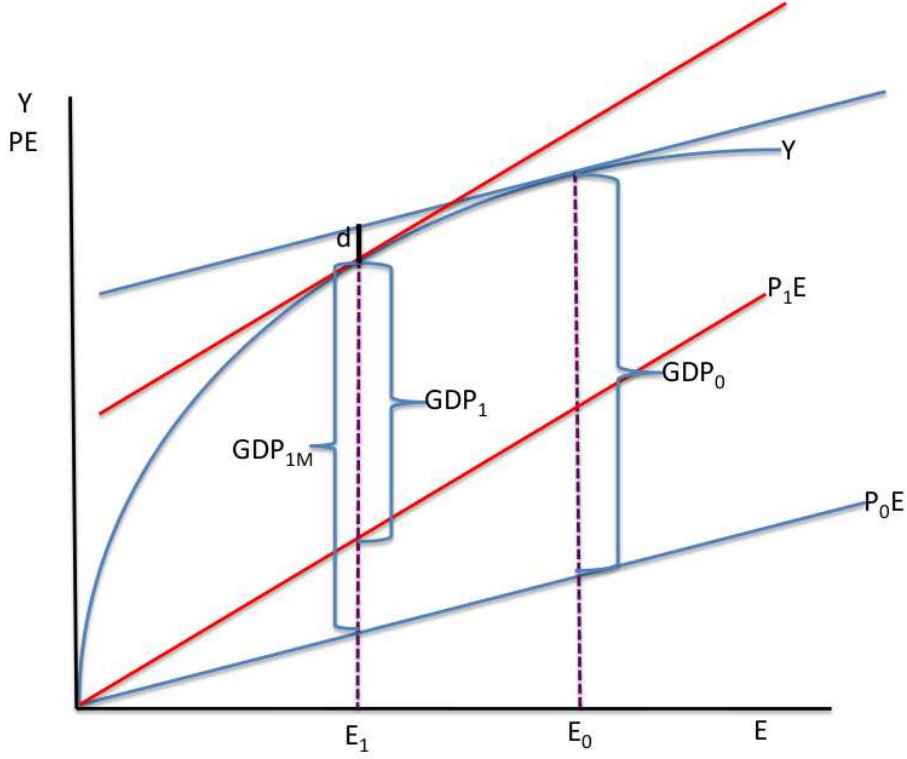
we call measured GDP (GDP_M) as

$$GDP_{Mt} = Y_t - \bar{p}E_t = Y_t \left[1 - \bar{p}p_t^{-\epsilon}\gamma^\epsilon \left(A_t^E\right)^{\epsilon-1}\right] = Y_t \left[1 - \gamma \left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon} \frac{\bar{p}}{p_t}\right]. \qquad (14)$$

Using the first equality in equation (14), the derivative of value added with respect to p_t is given by

$$\left(\frac{\partial Y_t}{\partial E_t} - \bar{p}\right) \frac{\partial E_t}{\partial p_t} = (p_t - \bar{p}) \frac{\partial E_t}{\partial p_t}.$$

Given that a firm optimally equates the marginal product of energy with the price, there is no effect of a marginal increase in the price of energy on measured value added if the price equals the baseline price of $\bar{p}$.

Since there is no effect on measured value added, there is no effect on the measured value for the Solow residual. Using equations (8) and (14), the measured Solow Residual

in the data can be expressed as

$$SR_{Mt} = \frac{GDP_{Mt}}{K_t^\alpha L_t^{1-\alpha}} = A_t \left(1-\gamma\right)^{\frac{\epsilon}{\epsilon-1}} \left[1 - \gamma \left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right]^{\frac{\epsilon}{1-\epsilon}} \left[1 - \gamma \left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon} \frac{\bar{p}}{p_t}\right].$$

The derivative of the Solow Residual with respect to the price of energy is zero if price equals its baseline value, implying no effect of small changes in the price of energy on the Solow Residual. However, this holds only for small changes in price. For large changes in price, concavity of production in equation (11) with respect to energy and linearity of the cost of energy with respect to energy, implies asymmetries, as we argue below.

Consider the measure of the Solow residual in the data using Figure 2. After the increase in price, the equilibrium quantity of energy is allowed to change, but the price is not. Therefore, the measured value for GDP after the price increase equals production at the new optimal energy input of E_1 less energy inputs valued at P_0, not at P_1, yielding a value for measured GDP as GDP_{1M}. GDP falls only by the vertical distance d. Therefore, the effect of the increase in the price of energy on GDP and on the Solow residual is much smaller in the data, and would be zero if there were no concavity in production with respect to imported energy. Indeed, an infinitesimally small price change, as with a derivative, implies no change in measured GDP.

Now compare the effect of a price increase and a price decrease on measured Solow residuals using Figure 3. We assume that the quantity of imported energy is initially optimal at E_0. As price changes, E moves in the opposite direction, and GDP is the difference between the new value of production and the new value of energy imports measured at the initial unchanged price. Therefore, a price decrease raises imported energy to E_1 and decreases GDP by d_1, whereas a price increase reduces imported energy to E_2 and reduces GDP by d_2. Consequently, any change in price reduces GDP for a given capital-labor composite, thereby reducing the measure of the Solow residual in the data. This is a strange asymmetry, with positive and negative oil price changes creating relatively similar negative responses of the measured values for GDP and the Solow residual.

An increase in the price of energy has very different effects on the Solow residual in terms of a numeraire, presented at the beginning of this section, and the measured Solow residual in constant dollars. The difference between the two measures lies in the way the cost of imported intermediate inputs are treated. When we compute "real" using a numeraire, the increase in energy price raises the cost of imported intermediate inputs, reducing GDP. When we use constant dollars, the increase in energy price does not count

16

Figure 3: Measured GDP and Energy Price Changes
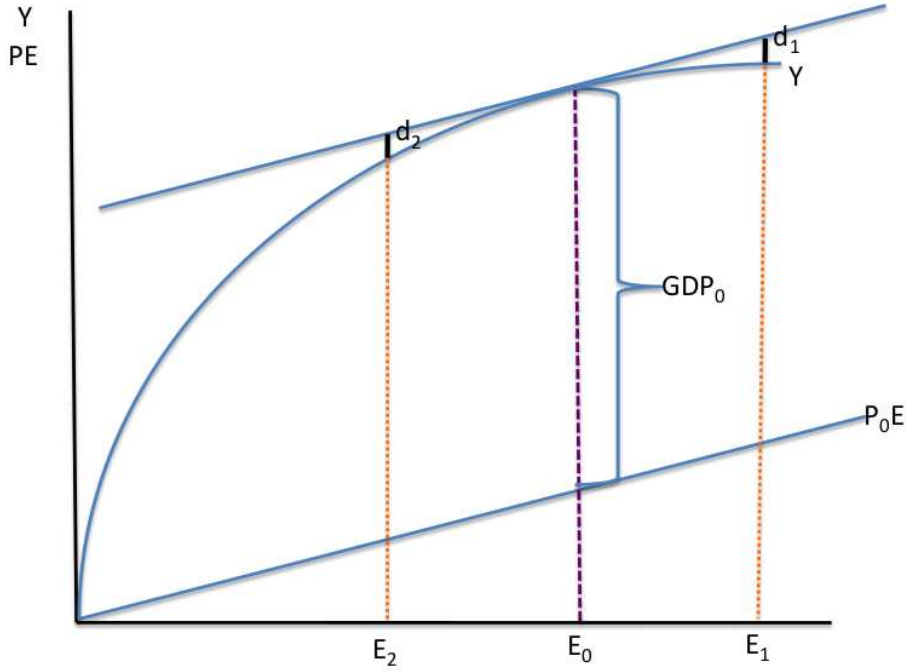
toward an increase in the cost of intermediate inputs since this cost must be measured in constant dollars. The fall in GDP due to higher cost of intermediate inputs is missing from the measured Solow residual in the data.

We can obtain an asymmetric effect of the measured Solow residual, in which we observe a larger negative response to an oil price increase than the positive response to the oil price decrease, if we introduce endogenous capacity utilization. Assume that capital is fixed in the short run, but that its utilization rate can vary. Modify equation (8) for production to include a capacity utilization term ($0 < z_t < 1$), yielding

$$Y_t = \left[(1 - \gamma) \left[A_t \left(z_t K_t \right)^\alpha L_t^{1-\alpha} \right]^{\frac{\epsilon-1}{\epsilon}} + \gamma \left[A_t^E E_t \right]^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$$

and assume, following Eichenbaum (1996) that capital depreciation is increasing in ca-

17

pacity utilization at an increasing rate. The firm chooses capacity utilization to maximize

$$\left[(1-\gamma)\left[A_t K_t^\alpha L_t^{1-\alpha}\right]^{\frac{\epsilon-1}{\epsilon}} + \gamma \left[A_t^E E_t\right]^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} - p_t E_t - \delta z_t^\phi K_t,$$

where $\phi > 1$, and δ is depreciation when $z_t = 1$.

First order conditions with respect to capacity utilization and energy price can be combined to yield

$$z_t = \left[\frac{\alpha K_t^\alpha L_t^{1-\alpha} A_t (1-\gamma)^{\frac{\epsilon}{\epsilon-1}} \left[1 - \gamma \left(\frac{p_t}{\gamma A_t^E}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{\delta \phi K_t}\right]^{\frac{1}{\phi-\alpha}} = \left[\frac{\alpha}{\delta \phi} \left(\frac{L_t}{K_t}\right)^{1-\alpha} SR_t\right]^{\frac{1}{\phi-\alpha}}. \tag{15}$$

Capacity utilization (z_t) responds to SR_t given by equation (13). The measure of the Solow residual in the data is irrelevant to the firm's optimizing behavior. Therefore, capacity utilization is a decreasing function of the price of oil since SR_t is a decreasing function of the price of oil.

Now consider how the oil price and capacity utilization interact to create asymmetries in the Solow residual once we allow for capacity utilization. The actual Solow residual, allowing for adjustment in capacity utilization (sr), can be expressed as

$$sr_t(z_t) = SR_t z_t^\alpha.$$

Endogenous capacity utilization magnifies the effect of the original shock, with both components of $sr(z_t)$, including SR_t and z_t^α, moving in the same direction.

The measured Solow residual, allowing for changes in capacity utilization, is analogously

$$sr_{Mt}(z_t) = SR_{Mt} z_t^\alpha, \tag{16}$$

where z_t from equation (15) depends on the actual value for the Solow residual, SR_t, not on its measured value. An increase in the price of oil creates a decrease in both SR_{Mt} and z_t^α, while a decrease in the price of oil creates a decrease in SR_{Mt} but an increase in z_t^α. Therefore, a decrease in the price of oil causes the two components of the measured Solow residual to move in opposite directions, while an increase moves them in the same direction, creating asymmetries. The asymmetries arise from concavity and are larger the larger the change in price.

These results imply that large exogenous increases in oil prices should have asymmetric negative effects on the measured Solow residual for countries with net energy imports. There should be no negative asymmetries for countries which are net exporters of energy.

5 Evidence on Determinants of Asymmetries

In this section, we reestimate the model to determine whether negative asymmetries remain, after conditioning on oil price increases and financial crises. The model requires a measure of the real oil price.[3] We measure the oil price by deflating the spot price of oil by the US CPI to obtain the real oil price.[4] We augment the baseline model (3) by a vector of, possibly lagged, explanatory variables x_t, including oil prices and financial crises, yielding

$$q_t = \beta_0 + \sum_{i=1}^{p} \beta_i q_{t-i} + \alpha' x_t + w_t, \tag{17}$$

where α is a vector of coefficients. The various conditional mean specifications are summarized in Tables 5 and 6 in the appendix.

5.1 The role of oil price shocks

Both the theoretical model of Section 4 and Hamilton (2011) imply that only large oil price increases should create large negative effects. Additionally, Kilian (2008) is concerned that the effect of oil price changes should be different depending whether the change is caused by supply or demand shocks. Supply disruptions are associated with large oil price increases, and by limiting our explanatory variable to large price increases, we largely avoid this problem.

Therefore, we require a nonlinear specification to capture the effect of unusually high oil prices. We follow Hamilton (2003, 2011) and include a variable that becomes effective only when the oil price attains a new 3-year high.[5] With X_t denoting the real oil price in period t, this variable is defined by

$$oil_t = \max(0, X_t - \max(X_{t-1}, \dots, X_{t-12})). \tag{18}$$

---

[3]Hamilton (2011) uses the nominal oil price, but since the CPI is not very volatile, the distinction should not matter much.

[4]We use the spot price of West Texas Intermediate from the FRED data base for the nominal oil price. The US CPI is the consumer price index for all urban consumers from BLS. We use this real measure of oil price for the US and other countries. An alternative for other countries would have been to convert the dollar price of oil into local currency price, using the exchange rate, and then deflate by the relevant foreign consumer price index. We did not take this route because this method would identify exchange rate crises as periods of large oil price increases.

[5]We also tried alternative functions of oil prices to capture the nonlinear effects, and results were similar.

Recall that the UK was a net energy importer for some periods and a net exporter for others. Therefore, we define a dummy variable, identifying the periods in which it was a net importer (1973Q1-1980Q2, 1988Q3-1992Q4 and 2003Q3-2011Q3). The dummy enters both as an intercept dummy and as a multiplier on oil_t, to allow the effect of oil prices to differ across the two regimes.

The estimation results of the models containing oil price shocks, defined by equation (18), are shown in Table 2. Statistics for skewness of the residuals and the estimated variance ratio demonstrate that the asymmetry is reduced for all countries that initially showed negative asymmetry, with the exception of Korea. For a subset of these countries, France and Italy, the null hypothesis of symmetry can no longer be rejected. For the UK we now have some evidence for positive asymmetry. For the US, the p-value of the symmetry hypothesis increases from 0.019 to 0.081. Thus, our empirical findings corroborate the implications of the theoretical model of Section 4. Large oil price increases are significant contributors to negative asymmetries in Solow residuals for net energy importers, and by implication, to asymmetries in business cycles for these countries. However, oil price shocks do not completely eliminate asymmetry for all countries. Even after conditioning on large oil price increases, Germany, Japan, Korea, and Switzerland continue to exhibit negative asymmetries. Hence, in the following we consider another potential source for asymmetry.

Table 2: Estimation results of the DSF model with oil

	Aus	Can	Fra	Ger	Ita	Jap	Kor	Nor	Swi	UK	US
μ_u	-0.57***	-0.35	-0.44***	0.54***	0.35**	0.62***	0.56***	-0.53***	0.57***	-0.43***	0.51***
σ_v	0.76***	0.84***	0.79***	0.79***	0.74***	0.70***	0.76***	0.81***	0.62***	0.80***	0.80***
skewness	0.352	0.108	0.220	-0.405	-0.139	-0.632	-0.747	0.241	-2.496	0.13	-0.105
LL sym.	-209.7	-199.7	-200.5	-209.0	-184.2	-207.1	-159.5	-209.8	-188.7	-203.9	-206.6
LL DSF	-207.6	-199.6	-199.1	-206.4	-183.8	-202.8	-154.0	-209.0	-172.0	-201.1	-205.6
LR stat.	4.178	0.268	2.754	5.074	0.734	8.601	11.01	1.577	33.32	5.572	1.957
p-val.	0.041	0.605	0.097	0.012	0.196	0.002	0.001	0.209	0.000	0.018	0.081
VR	0.559	0.175	0.308	0.468	0.226	0.788	0.546	0.439	0.855	0.288	0.407

Note: Table 2 reports the estimation results of model (4) defined in Section 2.1 consisting of an AR model with an error term composed of a one-sided exponentially distributed r.v. with parameter μ_u and a two-sided Gaussian r.v. with parameter σ_v. The conditional mean equation includes the nonlinear term defined in (18). The data are growth rates of quarterly Solow residuals calculated as explained in Section 3. Skewness refers to the sample skewness of the residuals from the model with asymmetry, LL sym and LL DSF are the log-likelihood values of the model restricted to symmetry and the general model respectively, LR stat is the log likelihood ratio statistics for the null hypothesis of a symmetrically distributed error term, and VR refers to the ratio of the estimated variances of the one-sided and the two sided innovations. When sample skewness is positive, estimation of model (4) fails, and we present results from model (5). ***, ** and * refer to significance at the 1%, 5% and 10% confidence level, respectively.

5.2 The role of financial crises

Another source of business cycle asymmetries could be extreme events such as the recent world-wide financial crisis. Barro (2006) included financial crises in his list of rare events. Therefore, we augment our model by conditioning on dummy variables representing financial crises. We date the recent financial crisis in the third quarter of 2008, the time of the Lehman Brothers collapse on September 15, 2008. We allowed for contemporaneous effects and lags based on the finding of a preliminary model selection. For the Asian countries, Japan and Korea, we added a dummy for the Asian crisis in the fourth quarter of 1997. For Switzerland we added another dummy for the first quarter of 1991 to account for a banking crisis following a sharp decline in real estate prices. Estimation results for the model including financial crises are contained in Table 3.

Conditioning on financial crises reduces negative asymmetries for all countries with initial negative asymmetry. These asymmetries are completely eliminated for France, Germany, and Switzerland, and we are unable to reject the null of positive asymmetries for Korea. The financial crisis dummy is not significant for Norway and Canada, but these countries did not have negative asymmetries initially. In Italy, Japan, Korea, the UK, and the US, negative asymmetries, measured by variance ratios, decrease, but we continue to reject the null of symmetry in favor of negative asymmetry for these countries. This evidence suggests that the financial crises alone are not responsible for all negative asymmetries observed in the data.

Finally, we allow conditioning on both oil price shocks and financial crises. The results are presented in Table 4. Evidence of negative asymmetry in the residuals has disappeared for all countries. Three countries with initial negative asymmetry, France, Korea, and the UK, now have significant positive asymmetry. This result suggests that a combination of large oil price increases and rare extreme events such as financial crises is responsible for the observed asymmetries in the Solow residual.

To summarize, for France, negative asymmetry can be completely eliminated with either the oil price variable or the financial crisis dummy. For Italy, the UK, and possibly for the US, depending on the desired critical value, the oil price variable completely eliminates evidence for negative asymmetry. For Germany, Switzerland, and Korea, the financial crisis dummy alone completely eliminates asymmetry. And, finally, for Japan and possibly the US, both oil prices and financial crises are needed to completely eliminate asymmetry. These results also confirm Barro's (2006) assumption that the asymmetric component of the Solow residual is determined by shocks which are large and infrequent

– in our sample large oil price increases and financial crises.

Table 3: Estimation results of the DSF model with crisis dummies

	Aus	Fra	Ger	Ita	Jap	Kor	Swi	UK	US
μ_u	-0.57***	-0.40***	-0.36	0.52***	0.49***	-0.59***	-0.24	0.55***	0.53***
σ_v	0.78***	0.84***	0.84***	0.68***	0.74***	0.56***	0.65***	0.80***	0.80***
skewness	0.325	0.009	0.071	-0.576	-0.404	0.868	0.111	-0.251	-0.120
LL sym.	-212.8	-205.1	-200.0	-192.2	-198.8	-139.4	-147.8	-206.7	-209.0
LL DSF	-210.8	-204.4	-199.8	-188.3	-196.8	-133.6	-147.7	-203.1	-207.9
LR stat.	4.077	1.482	0.358	7.942	4.026	11.51	0.171	7.278	2.272
p-val.	0.044	0.223	0.550	0.002	0.022	0.001	0.679	0.004	0.066
VR	0.531	0.223	0.185	0.583	0.440	1.104	0.142	0.480	0.432

Note: Table 3 reports the estimation results of model (4) defined in Section 2.1 consisting of an AR model with an error term composed of a one-sided exponentially distributed r.v. with parameter μ_u and a two-sided Gaussian r.v. with parameter σ_v. The conditional mean equation includes dummies for financial crises as described in the text. The data are growth rates of quarterly Solow residuals calculated as explained in Section 3. Skewness refers to the sample skewness of the residuals from the model with asymmetry, LL sym and LL DSF are the log-likelihood values of the model restricted to symmetry and the general model respectively, LR stat is the log likelihood ratio statistics for the null hypothesis of a symmetrically distributed error term, and VR refers to the ratio of the estimated variances of the one-sided and the two sided innovations. When sample skewness is positive, estimation of model (4) fails, and we present results from model (5). ***, ** and * refer to significance at the 1%, 5% and 10% confidence level, respectively.

Table 4: Estimation results of the DSF model with oil and crisis dummies

	Aus	Fra	Ger	Ita	Jap	Kor	Swi	UK	US
μ_u	-0.57***	-0.48***	-0.34	0.27	0.31**	-0.56***	-0.30	-0.36***	0.45***
σ_v	0.75***	0.72***	0.82***	0.74***	0.78***	0.57***	0.60***	0.69***	0.80***
skewness	0.366	0.436	0.038	-0.035	-0.121	0.836	0.243	0.129	-0.033
LL sym.	-207.9	-194.8	-196.3	-178.1	-189.0	-137.6	-142.9	-197.2	-202.9
LL DSF	-205.6	-192.5	-196.1	-178.0	-188.8	-132.4	-142.6	-195.5	-202.3
LR stat.	4.545	4.650	0.295	0.226	0.265	10.41	0.599	3.449	1.374
p-val.	0.033	0.031	0.587	0.317	0.303	0.001	0.439	0.063	0.121
VR	0.589	0.446	0.174	0.138	0.160	0.972	0.257	0.272	0.317

Note: Table 4 reports the estimation results of model (4) defined in Section 2.1 consisting of an AR model with an error term composed of a one-sided exponentially distributed r.v. with parameter μ_u and a two-sided Gaussian r.v. with parameter σ_v. The conditional mean equation includes the nonlinear term defined in (18) and dummies for financial crises as defined in the text. The data are growth rates of quarterly Solow residuals calculated as explained in Section 3. Skewness refers to the sample skewness of the residuals from the model with asymmetry, LL sym and LL DSF are the log-likelihood values of the model restricted to symmetry and the general model respectively, LR stat is the log likelihood ratio statistics for the null hypothesis of a symmetrically distributed error term, and VR refers to the ratio of the estimated variances of the one-sided and the two sided innovations. When sample skewness is positive, estimation of model (4) fails, and we present results from model (5). ***, ** and * refer to significance at the 1%, 5% and 10% confidence level, respectively.

6   Conclusion

We investigate business cycle asymmetry by estimating the degree of asymmetry present in total factor productivity, as measured by Solow residuals, for eleven OECD countries. Barro (2006) introduced the idea that total factor productivity could have two components, one which is distributed normal i.i.d., as in DSGE models, and one which has only negative realizations. Barro identified his asymmetric component with events which reduced real GDP by at least fifteen percent. In contrast to Barro, we estimate the extent of asymmetry present in Solow residuals using Stochastic Frontier Analysis.

We perform likelihood ratio tests to determine whether the Solow residual contains a negative asymmetric component and compute variance ratios to estimate the extent of the asymmetry. We find that eight of the eleven OECD countries in our sample have significant negative asymmetries. Additionally, we use the pattern of asymmetry across countries to understand its cause. All countries with significant negative asymmetries are net energy importers and all countries without are net energy exporters. This provides some support to Hamilton's (2011) hypothesis that large oil price changes have an asymmetric effect for energy importers.

Hamilton's hypothesis has been challenged both empirically and theoretically. In an attempt to answer the theoretical challenge, we present a theoretical model in which endogenous capacity utilization interacts with concavity in production to generate asymmetries for large oil price shocks. Then, we condition Solow residuals on Hamilton's net oil price increase variable and determine whether asymmetries are reduced. We find that asymmetries are reduced for all countries which initially had them, except for Korea, and that for some countries, there is no longer evidence of negative asymmetries. This provides additional support for the hypothesis that large oil price increases bear some responsibility for business cycle asymmetry, but some asymmetry does remain for some countries.

Finally, we introduce another cause of asymmetry, following Barro (2006), and condition on financial crises. Conditioning on financial crises alone reduces but does not eliminate asymmetry. However, conditioning on both the net oil price increase variable and financial crises completely eliminates any evidence of negative asymmetry in the Solow residuals. These results confirm Barro's assumption that asymmetric innovations to Solow residuals are due to large and infrequent shocks – in our sample large oil price shocks and financial crises. They also imply that understanding business cycles requires a model which admits large and infrequent shocks.

To conclude, a model which allows negative asymmetries in the Solow residual is a better fit for eight of eleven OECD countries than a model which requires symmetry. The countries with negative asymmetries all have a history of net oil imports. Conditioning on large oil price increases reduces or eliminates the asymmetries for all countries except Korea. Complete elimination of negative asymmetries for all countries requires the addition of financial crisis dummies. Our results empirically substantiate Barro's (2006) hypothesis that negative asymmetries in the Solow residual are due to extreme events, and add large oil price increases, proposed by Hamilton (2003, 2011), to Barro's list of extreme events.

A   Conditional mean equations

Table 5: Conditional mean equations 1

Aus	$q_t = 0.0018 - 0.1361q_{t-1} + w_t$
Aus-Oil	$q_t = 0.0637 - 0.1631 - 2.7323 + w_t$
Aus-Crisis	$q_t = 0.0121 - 0.141q_{t-1} - 1.5815cri_{t-1} + w_t$
Aus-Oil + Crisis	$q_t = 0.0755 - 0.1687q_{t-1} - 2.7768oil_{t-1} - 1.6501cri_{t-1} + w_t$
Can	$q_t = -0.0017 + 0.2467q_{t-1} + 0.277q_{t-3} + w_t$
Can-Oil	$q_t = 0.0467 + 0.2228q_{t-1} + 0.2825q_{t-3} - 2.1028oil_{t-2} + w_t$
Fra	$q_t = -0.0040 + 0.2597q_{t-1} + w_t$
Fra-Oil	$q_t = 0.0917 + 0.2630q_{t-1} - 4.1991oil_{t-3} + w_t$
Fra-Crisis	$q_t = 0.0164 + 0.2214q_{t-1} - 3.2105cri_{t-1} + w_t$
Fra-Oil + Crisis	$q_t = 0.108 + 0.2178q_{t-1} - 4.0676oil_{t-3} - 2.9506cri_{t-1} + w_t$
Ger	$q_t = w_t$
Ger-Oil	$q_t = 0.0668 - 2.9001oil_{t-3} + w_t$
Ger-Crisis	$q_t = 0.0488 - 2.7915cri_{t-1} - 4.4188cri_{t-2} + w_t$
Ger-Oil + Crisis	$q_t = 0.1019 - 2.48oil_{t-3} - 2.686cri_{t-1} - 3.941cri_{t-2} + w_t$
Ita	$q_t = -0.0175 + 0.3047q_{t-1} + w_t$
Ita-Oil	$q_t = 0.1108 + 0.1917q_{t-1} - 4.1726oil_{t-3} - 2.2682oil_{t-4} + w_t$
Ita-Crisis	$q_t = -0.0011 + 0.2163q_{t-1} - 2.0747cri_{t-1} - 3.2797cri_{t-2} + w_t$
Ita-Oil + Crisis	$q_t = 0.1266 + 0.1501q_{t-1} - 3.792oil_{t-3} - 2.2031oil_{t-4} - 1.5714cri_{t-1} - 2.5118cri_{t-2} + w_t$
Jap	$q_t = w_t$
Jap-Oil	$q_t = 0.1244 - 3.5755oil_t - 1.9212oil_{t-3} + w_t$
Jap-Crisis	$q_t = 0.0634 - 3.4232cri_{t-1} - 3.8860cri_{t-2} - 2.4627asi\_cri_{t-1} + w_t$
Jap-Oil + Crisis	$q_t = 0.1787 - 3.5868oil_t - 1.5569oil_{t-3} - 3.3728cri_{t-1} - 3.6023cri_{t-2} - 2.5119asi\_cri_{t-1} + w_t$

Note: Table 5 reports the estimated conditional mean equations for our dynamic stochastic frontier model defined by equations (2) to (5). The variable oil_t is defined in equation (18). The crisis dummies cri_t and asi_cri_t represent 2008Q3 and 1997Q4, respectively. All parameter estimates with the exception of the intercepts are significant at least at the 10% level of significance.

Table 6: Conditional mean equations 2

Kor	$q_t = w_t$
Kor-Oil	$q_t = 0.0630 - 4.0260 oil_{t-3} + w_t$
Kor-Crisis	$q_t = 0.0717 - 3.8978 cri_{t-1} - 4.2755 asi\_cri_{t-1} + w_t$
Kor-Oil + Crisis	$q_t = 0.1101 - 2.4806 oil_{t-3} - 3.7956 cri_{t-1} - 4.332 asi\_cri_{t-1} + w_t$
Nor	$q_t = -0.0023 - 0.2506 q_{t-1} + w_t$
Nor-Oil	$q_t = 0.0369 - 0.2497 q_{t-1} - 1.8121 oil_{t-3} + w_t$
Swi	$q_t = -0.0074 + 0.1840 q_{t-1} + w_t$
Swi-Oil	$q_t = -0.0424 + 0.1666 q_{t-1} + 1.9006 oil_{t-1} + w_t$
Swi-Crisis	$q_t = 0.0329 + 0.3775 q_{t-1} - 2.3414 cri_{t-1} - 6.676 swi\_cri_{t-1} + 3.1879 swi\_cri_{t-2} + w_t$
Swi-Oil + Crisis	$q_t = 0.0563 + 0.3640 q_{t-1} + 2.6688 oil_{t-2} - 4.0414 oil_{t-3} - 2.6698 cri_{t-1}$
	$\qquad -6.6495 swi\_cri_{t-1} + 3.7708 swi\_cri_{t-2} + w_t$
Uk	$q_t = w_t$
Uk-Oil	$q_t = 0.3382 - 0.5059 d\_import_t - 3.3368 oil_{t-3} d\_import_{t-3} + w_t$
Uk-Crisis	$q_t = 0.0626 - 2.5456 cri_t - 3.1156 cri_{t-1} + w_t$
Uk-Oil + Crisis	$q_t = 0.3080 - 0.3574 d\_import_t - 2.9683 oil_{t-3} d\_import_{t-3} - 1.8867 cri_t - 2.7954 cri_{t-1} + w_t$
US	$q_t = w_t$
US-Oil	$q_t = 0.1268 - 3.1609 oil_{t-2} - 1.6855 oil_{t-3} + w_t$
US-Crisis	$q_t = 0.0327 - 3.1611 cri_{t-1} + w_t$
US-Oil + Crisis	$q_t = 0.1321 - 2.6999 oil_{t-2} - 1.6587 oil_{t-3} - 2.5744 cri_{t-1} + w_t$

Note: Table 6 reports the estimated conditional mean equations for our dynamic stochastic frontier model defined by equations (2) to (5). The variable oil_t is defined in equation (18). The crisis dummies cri_t, asi_cri_t and swi_cri_t represent 2008Q3, 1997Q4 and 1991Q1, respectively. The dummy d_import_t takes on the value one for periods when the UK was a net energy importer, namely 1973Q1-1980Q2, 1988Q3-1992Q4 and 2003Q3-2011Q3. All parameter estimates with the exception of the intercepts are significant at least at the 10% level of significance.

References

Aigner, D. J., C. A. K. Lovel, and P. J. Schmidt (1977). Formulation and estimatoin of stochastic frontier production function models. *Journal of Econometrics 6*, 21–37.

Barro, R. (2006). Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics 121*, 823–866.

Barsky, R. B. and L. Kilian (2004). Oil and the macroeconomy since the 1970s. *Journal of Economic Perspectives 18*, 115–134.

Battese, G. E. and T. J. Coelli (1988). Prediction of firm-level technical efficiencies with a generalized frontier production function and panel data. *Journal of Econometrics 38*, 387–399.

Bolton, P. (2010). Energy imports and exports. Commons library standard note – UK parliament.

Clements, M. and H. M. Krolzig (2002). Can oil shocks explain asymmetries in the us business cycle? *Empirical Economics 27(2)*, 185–204.

Engemann, K. M., K. L. Kliesen, and M. T. Owyang (2011). Do oil shocks drive business cycles? some U.S. and international evidence. *Macroeconomic Dynamics 15*, 498–517.

Hamilton, J. D. (1983). Oil and the macroeconomy since world war II. *Journal of Political Economy 91*, 228–248.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica 57*, 357–384.

Hamilton, J. D. (2003). What is an oil shock? *Journal of Econometrics 113(2)*, 363–398.

Hamilton, J. D. (2011). Nonlinearities and the macroeconomic effects of oil prices. *Macroeconomic Dynamics 15(S3)*, 364–378.

Hassler, J., P. Krusell, and C. Olovsson (2012). Energy-saving technical change. NBER working paper 18456.

Kehoe, T. J. and K. J. Ruhl (2008). Are shocks to the terms of trade shocks to productivity? *Review of Economic Dynamics 11*, 804–819.

Kilian, L. (2008). A comparison of the effects of exogenous oil supply shocks on output and inflation in the G7 countries. *Journal of the European Economic Association 6(1)*, 78–121.

Kilian, L. and R. J. Vigfusson (2011a). Are the responses of the U.S. economy asymmetric in energy price increases and decreases? *Quantitative Economics*, forthcoming.

Kilian, L. and R. J. Vigfusson (2011b). Nonlinearities in the oil price-output relationship. *Macroeconomic Dynamics 15(S3)*, 337–363.

Kumbhakar, S. C. and C. A. K. Lovell (2000). *Stochastic Frontier Analysis*. Cambridge University Press.

Lee, L. F. (1993). Asymptotic distribution of the maximum likelihood estimator for a stochastic frontier function model with a singluar information matrix. *Econometric Theory 9*, 413–430.

McKay, A. and R. Reis (2008). The brevity and violence of contractions and expansions. *Journal of Monetary Economics 55(4)*, 738–751.

Meeusen, W. and J. van den Broek (1977). Efficiency estimation from Cobb-Douglas production functions with composed errors. *International Economic Review 18*, 435–444.

Mork, K. A. (1989). Oil and the macroeconomy when prices go up and down: An extension of Hamilton's results. *Journal of Political Economy 97(3)*, 740–744.

Murillo-Zamorano, L. R. (2004). Economic efficiency and frontier techniques. *Journal of Economic Surveys 18*, 33–77.

Raymond, J. E. and R. W. Rich (1997). Oil and the macroeconomy: A markov state-switching approach. *Journal of Money, Credit and Banking 29*, 193–213.