On the Value of Input-Efficiency, Capacity-Efficiency, and the Flexibility to Rebalance Them

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Abstract: A common characteristic of basic material manufacturers (which account for 85% of all industrial energy use) and of cleantech manufacturers is that they are price-takers in their input and output markets. Variability in those prices has implications for how much a manufacturer should invest in three fundamental types of process improvement. Input price variability reduces the value of improving *input-efficiency* (output produced per unit input) but increases that of *capacity-efficiency* (the rate at which a production facility can convert input into output). Output price variability has the same effects, if and only if the expected margin is large. When the expected margin is thin, output price variability increases the value of input-efficiency. Moreover, as the expected input cost rises, the value of input-efficiency decreases. A third type of process improvement is to develop *flexibility in input-efficiency versus capacity-efficiency* (the ability to respond to a rise in input cost or fall in output price by increasing input-efficiency at the expense of capacity-efficiency). The value of this flexibility decreases with variability in input and output prices, if and only if the expected margin is thin. Together, these results suggest that climate policy may reduce investment by basic material manufacturers in improving energy-efficiency.

1. Introduction and Overview

Every manufacturer strives to improve productivity in two key dimensions. The first, *input-efficiency*, is the amount of saleable output produced per unit input. The second, *capacity-efficiency*, is the rate at which a production facility can convert input into saleable output. This paper shows that variability in the market prices for a manufacturer's input and output has substantial implications for whether the manufacturer should focus on improving input-efficiency or capacity-efficiency. Input cost variability increases the value of improving capacity-efficiency, but reduces the value of improving input-efficiency. Output price variability has the same effect, if and only if the expected margin is large. When the expected margin is small, output price variability increases the value of improving input-efficiency. Furthermore (due to variability in input and output prices), an increase in the expected input cost can reduce the value of improving input-efficiency.

As an alternative to improving these two dimensions of efficiency, a manufacturer may develop flexibility to trade off the two types of efficiency. A manufacturer with *flexibility in input-efficiency versus capacity-efficiency* can adapt to a rise in input costs or fall in output prices by changing the production process to increase input-efficiency at the expense of capacity-efficiency. In so doing, the manufacturer gives up some output in order to reduce its variable cost of production. In contrast to the typical finding that the value of flexibility increases with variability in the business environment, this paper shows that the value of flexibility in input-efficiency versus capacity-efficiency decreases with variability in input and output prices, when margins are thin.

These insights are relevant to the wide swath of manufacturers that are price-takers in their input and output markets. They are particularly important for cleantech and basic material manufacturers, whose output prices and input costs are highly variable and uncertain, in large part due to volatility in energy prices and environmental policy. When input costs rise above output prices, firms suspend production–which is the phenomenon that drives our analytic results. (For examples of basic material (e.g., aluminum, ammonia) manufacturers suspending production in response to increased energy prices, see (U.S. Department of Energy 2007, Seay 2012). For examples of cleantech firms (e.g., biofuel, solar photovoltaic, and wind turbine manufacturers) suspending production due to changes in government policy, see (Wiser et al. 2007, Guzman et al. 2012, Mufson 2012).)

According to the Intergovernmental Panel on Climate Change (2007), one of the most important means to mitigate climate change is to improve energy-efficiency in the manufacturing of basic materials (chemicals, metals, minerals, paper, and petroleum products), which accounts for 85% of all industrial energy use and a quarter of all anthropogenic greenhouse gas emissions. However, our analysis suggests that a carbon tax or cap-and-trade system will discourage investment in energy-efficiency by incumbent basic material manufacturers. Therefore, eliminating emissions from basic material manufacturing may require cleantech manufacturers to displace those incumbents or supply them with "clean" energy.

The viability of cleantech firms will be enhanced by their making judicious investments in process improvement. This paper uses the example of a prominent cleantech manufacturer, Amyris, to illustrate that price variability has large–and directionally different–impacts on the value of improving input-efficiency and the value of improving capacity-efficiency. Furthermore, developing flexibility in input-efficiency versus capacity-efficiency is highly valuable, despite the fact that its value decreases with price variability.

Flexibility in input-efficiency versus capacity-efficiency is particularly important for many cleantech manufacturers. Amyris, for example, uses genetically-engineered yeast to ferment sugar into farnesene; farnesene is a precursor for various fuels and petrochemicals, and hence its price is tied to the price of oil. When the price of sugar rises relative to the price of oil, Amyris can switch to a strain of yeast that produces more farnesene per unit sugar input (increasing input-efficiency), but multiplies and produces the farnesene more slowly (reducing capacity-efficiency) (Lievense 2010). Many other cleantech firms use microorganisms to transform sugar or other biomass (from agricultural commodities, agricultural waste, or municipal waste) into fuels or chemicals. They all, characteristically, have the flexibility to increase input-efficiency at the expense of capacity-efficiency by modifying the microorganism or increasing the batch processing time (as in (Ata et al. 2012)). Alternatively, they may increase input-efficiency (output per unit *cost* of input) by using a cheaper, lower-quality input that requires more processing or reduces expected yield, and therefore reduces capacity-efficiency. Similarly, in molding the blades for a wind turbine, manufacturers have flexibility to use a cheaper resin that requires less heat but more time to infiltrate the mold and harden, which increases input-efficiency (blades per unit cost of input) but reduces capacity-efficiency (blades per unit time) (Stewart 2012). In another large category, solar photovoltaic manufacturers have the flexibility to increase input-efficiency (module output per unit of silicon input) by applying more thinly-sliced silicon, without reducing a module's light conversion efficiency. However, this increase in input-efficiency comes at the expense of capacity-efficiency (module output per unit time) because thinly-sliced silicon tends to break and require rework (Zuretti 2006).

Flexibility in input-efficiency versus capacity-efficiency has not previously been addressed in the economics or operations management literature, except in the model of a waste-to-energy firm in (Ata et al. 2012). Surveys of the literature on manufacturing flexibility are in (Sethi and Sethi 1990, Gerwin 1993, and Goyal and Netessine 2011).

This paper focus on the value of improving input-efficiency, capacity-efficiency and flexibility therein, but abstracts from the costs of doing so. For model-based and empirical research on how to improve input-efficiency or capacity-efficiency (and the associated costs) we refer the reader to (Carrillo and Gaimon 2000, Lapre et al. 2000, Gaimon 2008, Tanrisever et al. 2012) and the papers surveyed therein. The potential for flexibility in input-efficiency versus capacity-efficiency may be inherent in a production technology. However, that flexibility typically must be developed through investments in R&D or experimentation in the operating mode of a production facility, which, as shown in Carrillo and Gaimon (2000) and Terwiesch and Xu (2004), is costly.

Following Hicks' (1936) observation that an increase in the cost of an input will spur innovation to use that input more efficiently, current models for energy and climate policy analysis assume that firms will respond to an increase in energy prices by improving energy-efficiency or, for tractability, make energy-efficiency exogenous (Gillingham et al. 2008, 2012). Our paper helps to explain the contradictory empirical observation by Linn (2008) that when current or forecasted energy prices rise, incumbent firms in energy-intensive industries do *not* invest to improve energy-efficiency. An extensive literature documents that firms commonly fail to make seemingly profitable investments in energy-efficiency, and provides various explanations (see Jira and Lee 2012 and references therein). For example, Metcalf and Hasset (1993) explain that investment in energy-efficiency is irreversible, so uncertainty in future energy prices favors postponing such investments. We identify a different mechanism by which uncertainty in energy prices inhibits energy-efficiency.

2. Model Formulation

In our simple model, first, a manufacturer can choose to improve its process (by increasing capacity-efficiency, increasing input-efficiency, or developing flexibility in capacity-efficiency versus input-efficiency). Then, the manufacturer realizes its input and output prices, and chooses its production quantity (and mode, if it has developed flexibility) subject to its capacity constraint. The manufacturer is a price-taker in its input and output markets. That is, the manufacturer's cost per unit input C and selling price P per unit output are random variables that are independent of the manufacturer's input purchase and output sales quantities. We assume $C \sim$Normal(μ_c, σ_c^2), $P \sim$Normal(μ_p, σ_p^2), C and P have correlation ρ, and that for each random variable, the mean is sufficiently large relative to the standard deviation that the probability that the random variable is negative is negligible. Further, in expectation, the output price exceeds the input cost $\mu_p > \mu_c$. Let $\phi(c, p)$ denote the joint density of C and P.

In our baseline scenario, without loss of generality, we normalize both capacity and input-efficiency to 1, so that if the manufacturer produces, she produces one unit of output and consumes one unit of input. Baseline expected profit is

$$\Pi_O = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \max(p - c, 0)\phi(c, p)dpdc = \int_{-\infty}^{\infty}\int_{c}^{\infty} (p - c)\phi(c, p)dpdc. \tag{1}$$

The optimal policy is to produce when $p \geq c$ and otherwise to idle the production facility.

Through process improvement (e.g. cycle time reduction or elimination of down time), the manufacturer might increase its *capacity-efficiency* k above the baseline level 1, so that expected profit becomes

$$\Pi = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \max\left(k(p-c), 0\right)\phi(c,p)dpdc = \int_{-\infty}^{\infty}\int_{c}^{\infty} k(p-c)\phi(c,p)dpdc. \tag{2}$$

To increase *input-efficiency* i above the baseline level 1, the manufacturer might focus on increasing the expected yield of output per unit input (e.g., through quality management, to ensure that input is converted into saleable output rather than waste), so that expected profit becomes

$$\Pi = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \max\left((ip-c), 0\right)\phi(c,p)dpdc = \int_{-\infty}^{\infty}\int_{c/i}^{\infty} (ip-c)\phi(c,p)dpdc, \tag{3}$$

or the manufacturer might focus on reducing the amount of input required to produce a unit output (e.g., by improving the energy-efficiency of the production process), so that expected profit becomes

$$\Pi = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \max\left((p-c/i), 0\right)\phi(c,p)dpdc = \int_{-\infty}^{\infty}\int_{c/i}^{\infty} (p-c/i)\phi(c,p)dpdc. \tag{4}$$

We will investigate how variance and correlation in C and P influence the *value of improving capacity-efficiency* $\partial\Pi/\partial k$ and the *value of improving input-efficiency* $\partial\Pi/\partial i$ in the baseline scenario with $k = i = 1$.

Alternatively, the manufacturer may develop *flexibility in input-efficiency versus capacity-efficiency*, the ability to operate with higher input-efficiency $i > 1$ at the expense of reducing its capacity-efficiency to $k < 1$. The expected profit of a manufacturer with flexibility in input-efficiency versus capacity-efficiency is

$$\Pi_F = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \max\left(p-c, k(p-c/i), 0\right)\phi(c,p)dpdc. \tag{5}$$

(Equation (5) implicitly interprets input-efficiency as reducing the amount of input required to produce a unit of output as in (4). When input-efficiency is interpreted as increasing the expected yield as in (3), the formulation is identical, except that ki replaces k in (5). If $ki \geq 1$, then operating with higher input-efficiency always dominates operating at the nominal input-efficiency, so the issue of flexibility is irrelevant. If $ki < 1$, then the two formulations are equivalent.) Let $j = i(1-k)/(i-k)$, and observe that $j < i$. The optimal policy is: if $p > c/j$, use the nominal process with input-efficiency and capacity-efficiency of 1; if $p \in (c/i, c/j)$, then increase input-efficiency to $i > 1$ and reduce capacity-efficiency to $k < 1$; otherwise, idle the production facility.

Therefore, (5) simplifies to

$$\Pi_F = \int_{-\infty}^{\infty} \left(\int_{c/i}^{c/j} k(p - c/i)\phi(c,p)dp + \int_{c/j}^{\infty} (p - c)\phi(c,p)dp \right) dc.$$

We will investigate how variance in C and P influence the *value of flexibility in input-efficiency versus capacity-efficiency*

$$\Delta = \Pi_F - \Pi_O. \tag{6}$$

Our results and proofs hold when expected profit is maximized over a finite time horizon $[0, T]$, the input cost and output price are *stochastic*, the firm chooses its production rate (and mode, if it has flexibility) at each instant in time $t \in [0, T]$ and it does not hold inventory. To assess the impact of a process improvement on expected profit, we assume that the distribution of input cost and output price at each time $t \in [0, T]$ is bivariate normal with means μ_c and μ_p, correlation ρ, and variances proportional (not necessarily equal) to σ_c^2 and σ_p^2. The input cost and output price processes could be zero-drift Brownian motion or fractional Brownian motion, for example, or realized at time zero and constant thereafter due to a contract. (The standard assumption in the literature on commodity prices is that prices follow a mean-reverting or geometric Brownian motion, implying a lognormal distribution at each point in time. Our model is consistent with this literature because a lognormal distribution is well approximated by a normal distribution whenever its mean is much larger than its standard deviation[1], as we have assumed regarding the input cost C and output price P.) Assuming that the input cost and output price evolve according to Brownian motion, our results hold, qualitatively, when the firm decides periodically whether or not to run a batch process, and our results regarding the impact of changes in μ_c and σ_c hold, qualitatively, when the firm can hold inventory at some constant cost per unit, per unit time.

[1]Let the mean of a lognormal random variable approach infinity, while fixing its standard deviation at any strictly positive level. Its skew and excess kurtosis converge to zero. Hence when the mean is large, one can approximate the first four significant moment (hence, shape) characteristics of the lognormal with those of a normal distribution (mean, variance, and zero skew and excess kurtosis).

3. Results

One might think that as an input becomes more costly, it becomes more important to use that input more efficiently. Proposition 1 reveals that under a broad set of conditions, that conjecture is incorrect.

Proposition 1 *The value of improving input-efficiency in* (3) *(increasing yield) decreases with the expected input cost*

$$(\partial^2/\partial i \partial \mu_c)\Pi < 0. \tag{7}$$

The value of improving input-efficiency in (4) *(reducing input requirements) decreases with the expected input cost if and only if*

$$\text{Prob}(P - C \geq 0) < \lim_{i \to 1}(\partial/\partial i)\, \text{Prob}(P - C/i \geq 0). \tag{8}$$

Inequality (8) tends to hold when the expected margin $m \equiv \mu_p - \mu_c$ is sufficiently small. More precisely, when the correlation between input cost and output price $\rho = 0$, inequality (8) holds if and only if the expected margin $m < \overline{m}$, where $\overline{m} > 0$. When $\rho \neq 0$, (8) holds if $m < \overline{m}$, where $\overline{m} > 0$ if the relatively mild condition that expected input cost is sufficiently large relative to the standard deviation of the contribution margin $\mu_c > \sqrt{(\sigma_c^2 - 2\rho\sigma_c\sigma_p + \sigma_p^2)\pi/2}$ is satisfied; (8) is violated when the expected margin m is sufficiently large.

The intuition for Proposition 1 is as follows. Consider the setting in which input-efficiency reduces the input requirement, which for concreteness we interpret as improving energy-efficiency. An increase in the expected cost of the energy input has two opposing effects on the value of improving energy-efficiency. First, an increase in the cost of energy decreases the probability that the manufacturer operates. Because improving energy-efficiency is of value only in the event the manufacturer operates, decreasing this probability decreases the value of energy-efficiency. Second, in the event that the manufacturer operates, increasing the cost of energy makes energy-efficiency more valuable. The first (negative) effect dominates the second to the extent that the manufacturer expected margin m is small, so that the probability of operating is small and more sensitive to an increase in the cost of energy. In the setting in which input-efficiency increases the yield, only the first effect–increasing the input cost reduces the probability that the manufacturer operates–is at work, so increasing the input cost reduces the value of improving input-efficiency.

For an input-efficiency improvement at Amyris that takes the form of (4), we find that the company's thin margins imply that (8) holds, as explained in the numerical example section below. Thus, Proposition 1 implies that when the expected cost of sugar rises, Amyris should invest *less* in process improvement efforts aimed at using less sugar.

Proposition 2 characterizes how variability and correlation in input and output prices affect the values of improving capacity- and input-efficiency. While, for completeness, the Internet Appendix provides a complete characterization for correlation $\rho \in (-1, 1)$, Proposition 2 focuses on the case where the correlation

$$\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c), \tag{9}$$

which means that variability in the contribution margin $P - C$ increases with feedstock cost variability σ_c and with output price variability σ_p, and decreases with correlation. This condition is satisfied in the numerical example based on Amyris below.

Proposition 2 *Suppose* (9)*. The value of improving capacity-efficiency increases with input cost variability*

$$(\partial^2/\partial k \partial \sigma_c)\Pi > 0 \tag{10}$$

and the value of improving input-efficiency decreases with input cost variability

$$(\partial^2/\partial i \partial \sigma_c)\Pi < 0. \tag{11}$$

The value of improving capacity-efficiency increases with output price variability

$$(\partial^2/\partial k \partial \sigma_p)\Pi > 0. \tag{12}$$

The value of improving input-efficiency decreases with output price variability

$$(\partial^2/\partial i \partial \sigma_p)\Pi < 0 \tag{13}$$

if and only if the expected margin is large $m > \overline{m}$*, where* $\overline{m} \in (0, \infty)$*. The value of improving* *capacity-efficiency decreases with correlation between the input cost and the output price*

$$(\partial^2/\partial k \partial \rho)\Pi < 0. \tag{14}$$

The value of improving input-efficiency increases with that correlation

$$(\partial^2/\partial i \partial \rho)\Pi > 0 \tag{15}$$

if and only if the expected margin $m > \underline{m}$.

The closed form expressions for $\overline{m}$ and $\underline{m}$ are evident in the proof of Proposition 2, and give us the following observations. When input-efficiency reduces the input requirement as in (4), $\underline{m} < 0$, meaning that (15) always holds. When input-efficiency increases the output yield as in (3), the following hold: The thresholds $\overline{m}$ and $\underline{m}$ are strictly positive and decreasing in μ_c, implying that (13) and (15) tend to hold when the expected output price μ_c is large. As $\sigma_c \to 0$, $\overline{m} \to 0$ and $\underline{m} \to 0$, which implies that (13) and (15) hold when the variability in the input cost σ_c is sufficiently small. When $\rho = 0$, $\overline{m} = \underline{m} = \left[\sqrt{(\sigma_c^2 + \sigma_p^2)[\mu_c^2(\sigma_c^2 + \sigma_p^2) + 4\sigma_c^4]} - \mu_c(\sigma_c^2 + \sigma_p^2)\right]/(2\sigma_c^2)$, which is increasing in σ_p, implying that (13) and (15) tend to hold when input price variability σ_p is small. (The results regarding the impact of μ_c and σ_p on $\overline{m}$ also apply when input efficiency reduces the input requirement.)

Proposition 2 reveals that variability in input prices has the opposite effect on the value of improving input-efficiency as on the value of improving capacity-efficiency. Variability in the output price and correlation also do so, provided that the expected margin m is large enough.

The results and intuition regarding capacity-efficiency are straightforward. The manufacturer has the option to adjust her production decision in response to the realized input cost and output price. As is well known in finance, the value of such an option increases with variability in the margin. Capacity-efficiency is a multiplier on expected profit, so an increase in capacity-efficiency becomes more valuable with variability in the margin and hence (when correlation is not too large) with variability in input cost and output price, and with reduced correlation.

The results and intuition regarding the value of improving input-efficiency are sharpest when the expected margin is large $m >> 0$. We begin with this case before turning to how the results change when the margin is small. The primary benefit of increasing input-efficiency from the nominal value $i = 1$ is that doing so increases profit when it it economical to produce, an event that occurs with Prob$(P - C > 0)$. When there is little variability in the contribution margin, it is almost always economical to produce ($m >> 0$ implies Prob$(P - C > 0) \approx 1$), and the manufacturer benefits the most from increasing input-efficiency. As variability in the contribution margin increases, the probability that the manufacturer will be able to use the improved input-efficiency decreases, as does the value of this efficiency. Consequently, improved input-efficiency becomes less valuable with variability in input cost and output price (when the correlation is not too large), and with reduced correlation.

The results change when the expected margin is narrow for two reasons. First, the effect described above diminishes. In the limiting case with a zero expected margin, the probability that it is economical to produce is unaffected by the variability in the contribution margin. Consequently, a secondary benefit of increasing input-efficiency–doing so increases the probability that it will be economical to produce–comes into play. This secondary effect can offset the primary effect, such that when the expected margin is small, input-efficiency becomes more valuable with variability in the output price, and with reduced correlation.

The managerial contribution of Proposition 2 is to characterize how a manufacturer should respond in terms of efficiency improvement efforts to changes in the variability and correlation in input and output prices. When input cost variability increases, a manufacturer should focus more effort on improving capacity-efficiency and less effort on improving input-efficiency. A large-margin manufacturer should respond in the same way to an increase in variability in the output price or a decrease in the correlation between the input cost and output price. A caveat is that improvements in input-efficiency and capacity-efficiency are complementary, so any improvement in capacity-efficiency will increase the value of improving input-efficiency.

Flexibility in Input-efficiency versus Capacity-efficiency

A manufacturer that develops flexibility in input-efficiency versus capacity-efficiency can adapt to a rise in input costs or fall in output prices by increasing input-efficiency at the expense of capacity-efficiency. In many settings, flexibility is a tool to cope with variability in the external environment, and so investing in flexibility is sensible provided that the variability in the external environment is sufficiently high (Tombak and DeMeyer 1988, Mills 1984, Anupindi and Jiang 2008, Chod and Rudi 2005, Goyal and Netessine 2011). Proposition 3 identifies circumstances under which precisely the opposite is true regarding flexibility in input-efficiency versus capacity-efficiency.

Before stating the result, to build intuition, it is helpful to note under what price and cost realizations the flexibility in input-efficiency versus capacity-efficiency is of value. It is of no value when the input cost is very low or very high relative to the output price. If the input cost is very high $c > ip$, then it is not economical to produce at all. If the input cost is low $c < jp$, then it is optimal to squeeze every possible unit of output from the production facility by using as much of the input as possible (i.e., operating at the nominal input-efficiency). Flexibility increases the manufacturer's profit only when the input cost is moderately high relative to the output price $c \in (jp, ip)$. Then, it is attractive to use the input more efficiently, at the expense of sacrificing some output.

Proposition 3 *There exists $\overline{m} > \mu_c(1-j)/(1+j)$ such that if $\sigma_p = 0$, then the value of flexibility in input-efficiency versus capacity-efficiency decreases with input cost variability*

$$(\partial/\partial\sigma_c)\Delta < 0 \tag{16}$$

if and only if the manufacturer's expected margin $m < \overline{m}$. There exists $\overline{m} > \mu_c(1-j)/(2j)$ such that if $\sigma_c = 0$, then the value of flexibility in input-efficiency versus capacity-efficiency decreases with output price variability

$$(\partial/\partial\sigma_p)\Delta < 0 \tag{17}$$

if and only if the manufacturer's expected margin $m < \overline{m}$.

The value of flexibility in input-efficiency versus capacity-efficiency decreases with variability in the input cost or output price if and only if the expected margin is sufficiently small. Proposition 3 formalizes this result for the case variability is present on only one side (input cost or output price), but the result continues to hold when there is variability on the other side, provided that it is sufficiently small.

First, we discuss the intuition for why the value of flexibility is decreasing in input cost variability. This result occurs when the expected margin (and the variability in the output price) is not too large. The driving force behind the intuition is that flexibility in input-efficiency versus capacity-efficiency is of value only when the input cost is moderate $c \in (jp, ip)$. When the expected margin is small, the effect of increasing variability in the input cost is to make moderate realizations of the input cost less likely and extreme realizations more likely. This shifts probability mass away from the realizations where flexibility in input-efficiency versus capacity-efficiency creates value, which reduces the expected value of this flexibility. (In contrast, when the expected margin is large, the effect of increasing variability in the input cost is to make moderate realizations of the input cost more likely, which increases the expected value of flexibility.) The intuition for the impact of output price variability on the value of flexibility in input-efficiency versus capacity-efficiency parallels the intuition for the impact of input cost variability.

We conclude that the manufacturer's margin plays a critical role in determining whether increased variability increases the value of flexibility in input-efficiency versus capacity-efficiency. When variability is primarily on one side, the results are particularly sharp. Increased variability in that dimension increases the value of developing flexibility for a rich-margin manufacturer, but reduces the value for a thin-margin manufacturer. Because many price-taking manufacturers have relatively thin margins, the latter observation is especially relevant.

Numerical Example: Amyris

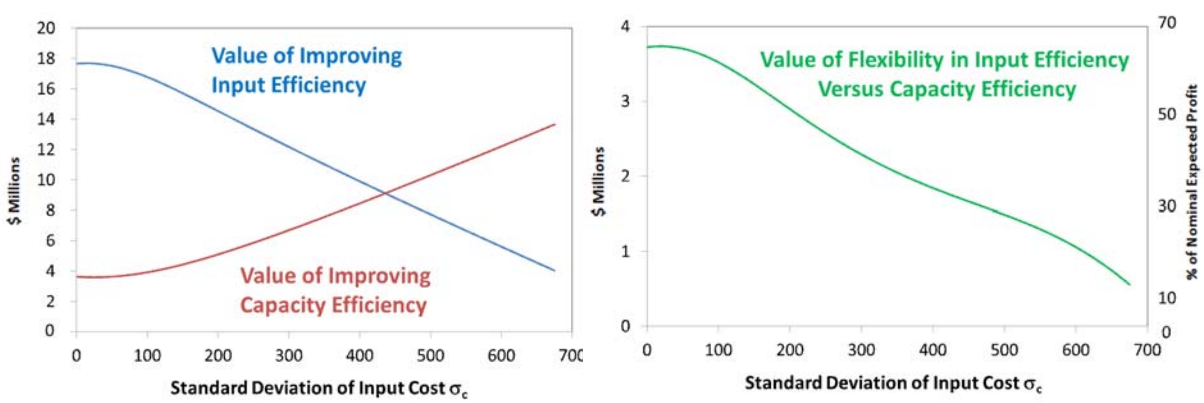
This section shows that for Amyris, the effects characterized in Propositions 1, 2 and 3 are large in magnitude.

Amyris' production facility has nominal production capacity of 50,000 tons of farnesene per year, at an input-efficiency of 1 ton of farnesene per 20 tons of crushed sugarcane. The output price for farnesene P in \$/ton is equal to the price of West Texas Intermediate (WTI) crude oil in \$/bbl, multiplied by 8.063. A ton of sugarcane contains 135 kg of raw sugar. (Amyris 2011) A sugarcane farmer captures 60% of the value of that raw sugar (Consecana 2006). Therefore we assume that the input price C in \$/(20 tons sugarcane) is the price of raw sugar in \$/kg multiplied by (135/0.6)/20; this is the variable cost of producing a ton of farnesene in the nominal mode of production. We also assume that the price of WTI oil and the price of raw sugar have a bivariate normal distribution with mean, standard deviation, and correlation calculated from the weekly prices for WTI oil and raw sugar #11 reported in the Global Financial Data database for the five years from June 2, 2007 through June 1, 2012. Hence the output price P and input cost C have a joint normal distribution with μ_p=685.5, σ_p=168.7, μ_c=675.3, σ_c=253.4 and ρ=0.1415.

The figure shows the value of improving capacity-efficiency ($\partial\Pi/\partial k$ in (2)) and the value of improving input-efficiency ($\partial\Pi/\partial i$ in (4)) as the standard deviation of the input cost σ_c varies from 0 to μ_c (left panel). Consistent with Proposition 2, variability in the input cost decreases the value of improving input-efficiency and increases that of capacity-efficiency. In contrast to the effect of input cost variability depicted in the figure, output price variability increases the value of improving input-efficiency and that of capacity-efficiency; this is consistent with Proposition 2 because Amyris' expected margin is thin $m < \overline{m}$. The impact of output price variability on value of improving capacity-efficiency is pronounced: at the "true" σ_p=168.7, the value of improving input-efficiency is 7% higher and the value of improving capacity-efficiency is 11% higher than in a scenario with no variability in output prices σ_p=0. The impact of input cost variability is even more substantial: at the "true" σ_c=253.4, the value of improving input-efficiency is 25% lower and the value of improving capacity-efficiency is 63% higher than in a scenario with no variability in input costs σ_c=0. The large magnitude of the effect of variability in input costs and output prices shows the importance of accounting for such variability in deciding how much to invest in different forms of process improvement.

The value of improving input-efficiency decreases in the expected input cost, which is consistent with Proposition 1 because Amyris' thin expected margin implies that inequality (8) holds. The impact is large in magnitude: at the “true” μ_c=675.3, the value of improving input-efficiency is 208% higher that at an expected input cost of μ_c=1008, the average cost experienced in the last year of the five-year time frame.

The right panel depicts the value of flexibility in input-efficiency versus capacity-efficiency (Δ in (6)) as the standard deviation of the input cost σ_c varies from 0 to μ_c. We assume that Amyris has the flexibility to improve input-efficiency to i =1.25 by reducing its capacity-efficiency to k =0.9. This is “in the ballpark" but smaller than the magnitude of flexibility reported for other industrial processes that rely on metabolism by microorganisms (Bouallagui et al. 2003). Nevertheless, the value of that flexibility is large in magnitude. At the “true” μ_c=253.4, it increases expected profit by 43%. Consistent with Proposition 3, variability in the input cost decreases the value of flexibility. The impact is substantial. At the “true” σ_c=253.4, the value of flexibility is 31% lower than in a scenario with no variability σ_c=0.



Energy-Efficiency and CO_2 Emissions in Basic Material Manufacturing

A tax on greenhouse gas emissions (or any other policy that increases the cost of fossil fuels) will tend to *reduce* investment in improving energy-efficiency in basic material manufacturing. This observation follows from Proposition 1, because the tax will increase the expected energy cost and reduce expected margins, which are already thin for many basic material manufacturers.

In addition, any uncertainty and variability in the cost of greenhouse gas emissions (inherent in a cap-and-trade system, for example) will tend to *further reduce* investment in improving energy-efficiency in basic material manufacturing. Variability in the cost of emissions translates into variability in energy costs for basic material manufacturers, which Proposition 2 suggests will reduce their investment in energy-efficiency.

Moreover, for basic material manufacturers with thin margins, Proposition 3 shows that variability will tend to *reduce* investment in developing flexibility to operate with high energy-efficiency by sacrificing some capacity-efficiency. A manufacturer without that flexibility uses strictly more energy per unit output, in expectation.

An important caveat is that our analysis relies on the assumption that a firm does not produce when the input cost exceeds the output price. In reality, a firm might do so to meet a commitment to a customer, to build inventory (anticipating a higher selling price in future), due to setup costs in starting and stopping production, or because the output price drops during a batch process[2]. A firm that is more likely to operate has a greater motivation to improve its energy-efficiency as energy become more costly. However, a setup cost or batch processing time could motivate a firm to postpone the start of production until the output price substantially exceeds the input cost (as in the example of ammonia manufacturing (Seay 2012)) and thus reinforce the negative impact of a carbon tax on investment in energy-efficiency.

[2] In efforts to increase output per unit time, reducing the batch processing time is more attractive than increasing the output yield per batch in that reducing the batch time reduces the firm's exposure to a drop in the output price. We thank an anonymous referee for this observation.

In conclusion, our results suggest that a carbon tax or cap-and-trade system could backfire by reducing investment in energy-efficiency by incumbent basic material manufacturers. Furthermore, a tax that moved inversely with the market price of a fossil fuel (to reduce the variability in the effective price of that fuel) would be less of a deterrent to investment in energy-efficiency.

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Appendix

Proof of Proposition 1: For (3), $(\partial/\partial\mu_c)\Pi = -\operatorname{Prob}(iP - C \geq 0)$. Because by assumption the probability that P is negative is negligible, $\operatorname{Prob}(iP - C \geq 0)$ is strictly increasing in i, which implies (7). For (4), $(\partial/\partial\mu_c)\Pi = -\operatorname{Prob}(P - C/i \geq 0)/i$ and

$$\lim_{i\to 1}(\partial^2/\partial i\partial\mu_c)\Pi = \operatorname{Prob}(P - C \geq 0) - \lim_{i\to 1}(\partial/\partial i)\operatorname{Prob}(P - C/i \geq 0). \quad \blacksquare$$

We prove Proposition 2 under a very general formulation of input efficiency, which subsumes formulations (3) and (4) as extreme cases. Specifically, under input-efficiency $i \geq 1$, the manufacturer's production process consumes $f(i) \leq 1$ units of input to produce ("yield") $y(i) \geq 1$ units of output, where $y(\cdot)$ is increasing and $f(\cdot)$ is decreasing, and at least one is strictly so. At the nominal input-efficiency of $i = 1$, $y(1) = f(1) = 1$. The manufacturer's expected profit is

$$\Pi = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \max\left(y(i)p - f(i)c, 0\right)\phi(c,p)dpdc = \int_{-\infty}^{\infty}\int_{cf(i)/y(i)}^{\infty} (y(i)p - f(i)c)\phi(c,p)dpdc, \quad (18)$$

With $y(i) = i$ and $f(i) = 1$, (18) simplifies to (3); with $y(i) = 1$ and $f(i) = 1/i$, (18) simplifies to (4). For use in the proof of Proposition 2, define

$$\begin{aligned}
\alpha(\mu_1,\mu_2,\sigma_1,\sigma_2) &= -\mu_2^2\sigma_1(\sigma_1-\rho\sigma_2)^2 + \mu_1\mu_2(\sigma_1-\rho\sigma_2)(\sigma_1^2-\sigma_2^2) \\
&\quad + \mu_1^2\sigma_2[(1+\rho^2)\sigma_1\sigma_2 - \rho(\sigma_1^2+\sigma_2^2)] - (1-\rho^2)\sigma_1\sigma_2^2(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2) \\
\beta(\mu_1,\mu_2,\sigma_1,\sigma_2) &= (\sigma_1-\rho\sigma_2)(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)^2 - \alpha(\mu_1,\mu_2,\sigma_1,\sigma_2) \\
\gamma(\sigma_1,\sigma_2) &= \sigma_1^3 - 3\rho\sigma_1^2\sigma_2 + (2+\rho^2)\sigma_1\sigma_2^2 - \rho\sigma_2^3 \\
\underline{\rho}(\sigma_1,\sigma_2) &= \left(3\sigma_1^2 + \sigma_2^2 - \sqrt{5\sigma_1^4 - 2\sigma_1^2\sigma_2^2 + \sigma_2^4}\right)/(2\sigma_1\sigma_2) \\
s &= \sqrt{\sigma_c^2 - 2\rho\sigma_c\sigma_p + \sigma_p^2} \\
\lambda(m) &= [(\mu_c+m)^2 - s^2]\sigma_c(\sigma_c-\rho\sigma_p) - (\mu_c+m)\mu_c(\sigma_c^2-\sigma_p^2) + \mu_c^2(\rho\sigma_c-\sigma_p)\sigma_p \\
\theta(m) &= -\lambda(m) - s^2.
\end{aligned}$$

For use in the proofs of Propositions 2 and 3, let $\psi(X)$ denote the density of the random variable X evaluated at 0.

Proof of Proposition 2: First, we establish the results involving σ_c. For (2), observe that $\lim_{k\to 1}(\partial^2/\partial k\partial\sigma_c)\Pi = (\sigma_c - \rho\sigma_p)\psi(P-C) > 0$, where the inequality follows because $\rho < \sigma_c/\sigma_p$; this establishes (10). For (18), observe that $(\partial/\partial\sigma_c)\Pi = f(i)[f(i)\sigma_c - y(i)\rho\sigma_p]\psi(y(i)P - f(i)C)$. Further, $\lim_{i\to 1}(\partial^2/\partial i\partial\sigma_c)\Pi = \tau_c(m)\psi(P-C)/s^4$, where

$$\tau_c(m) = \alpha(\mu_c,\mu_c+m,\sigma_c,\sigma_p)y'(1) + \beta(\mu_c,\mu_c+m,\sigma_c,\sigma_p)f'(1).$$

Because $(\partial^2/\partial m^2)\tau_c(m) = -2\sigma_c(\sigma_c - \rho\sigma_p)^2(y'(1) - f'(1)) < 0$ and $\lim_{m\to 0}(\partial/\partial m)\tau_c(m) = -\mu_c(\sigma_c - \rho\sigma_p)s^2(y'(1) - f'(1)) < 0$, $\tau_c(m)$ is concave decreasing in m on $m \geq 0$. To establish (11) it is sufficient to show that

$$\tau_c(0) < 0 \tag{19}$$

for $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$, because $\tau_c(0) \geq \tau_c(m)$ for $m \geq 0$. Note that

$$\tau_c(0) = s^2\gamma(\sigma_c, \sigma_p)f'(1) - (1 - \rho^2)\sigma_c\sigma_p^2 y'(1).$$

Therefore, to establish (19), it is sufficient to show that

$$\gamma(\sigma_c, \sigma_p) > 0 \tag{20}$$

for $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$. It is straightforward to show that (20) holds if and only if $\rho < \underline{\rho}(\sigma_c, \sigma_p)$. Further, if $\sigma_c < \sigma_p$, then $\underline{\rho}(\sigma_c, \sigma_p) \in (\sigma_c/\sigma_p, 1)$; otherwise $\rho_c \geq 1$. Therefore, because $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c) \leq \underline{\rho}(\sigma_c, \sigma_p)$, (20) and hence (19) hold.

Second, we establish the results involving σ_p. For (2), observe that $\lim_{k\to 1}(\partial^2/\partial k\partial\sigma_p)\Pi = (\sigma_p - \rho\sigma_c)\psi(P - C) > 0$, where the inequality follows because $\rho < \sigma_p/\sigma_c$; this establishes (12). For (18), observe that $(\partial/\partial\sigma_p)\Pi = y(i)[y(i)\sigma_p - f(i)\rho\sigma_c]\psi(y(i)P - f(i)C)$. Further, $\lim_{i\to 1}(\partial^2/\partial i\partial\sigma_p)\Pi = \tau_p(m)\psi(P - C)/s^4$, where

$$\tau_p(m) = \beta(\mu_c + m, \mu_c, \sigma_p, \sigma_c)y'(1) + \alpha(\mu_c + m, \mu_c, \sigma_p, \sigma_c)f'(1).$$

Note that $(\partial^2/\partial m^2)\tau_p(m) = -2\sigma_c[(1+\rho^2)\sigma_c\sigma_p - \rho(\sigma_c^2 + \sigma_p^2)](y'(1) - f'(1)) < 0$ and $\lim_{m\to 0}(\partial/\partial m)\tau_c(m) = -\mu_c(\sigma_p - \rho\sigma_c)s^2(y'(1) - f'(1)) < 0$, where the inequalities hold because $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$. Therefore, $\tau_p(m)$ is concave decreasing in m on $m \geq 0$. Further, $\lim_{m\to\infty}\tau_p(m) < 0$. To establish that (13) holds if and only if $m > \overline{m}$, where $\overline{m} \in (0, \infty)$ it is sufficient to show that

$$\tau_p(0) > 0 \tag{21}$$

for $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$. Note that

$$\tau_p(0) = s^2\gamma(\sigma_p, \sigma_c)y'(1) - (1 - \rho^2)\sigma_c\sigma_p^2 f'(1).$$

Therefore, to establish (21), it is sufficient to show that

$$\gamma(\sigma_p, \sigma_c) > 0 \tag{22}$$

for $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$. It is straightforward to show that (22) holds if and only if $\rho < \underline{\rho}(\sigma_p, \sigma_c)$. Further, if $\sigma_c > \sigma_p$, then $\underline{\rho}(\sigma_p, \sigma_c) \in (\sigma_p/\sigma_c, 1)$; otherwise $\underline{\rho}(\sigma_p, \sigma_c) \geq 1$. Therefore, because $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c) \leq \underline{\rho}(\sigma_p, \sigma_c)$, (22) and hence (21) hold. (Note that because $\tau_p(m)$ is quadratic in m and $\overline{m}$ is the larger root of $\tau_p(m) = 0$, $\overline{m}$ is readily available in closed form.)

Third, we establish the results involving ρ. For (2), observe that $\lim_{k\to 1}(\partial^2/\partial k\partial\rho)\Pi = -\sigma_c\sigma_p\psi(P - C) < 0$, which establishes (14). For (18), observe that $(\partial/\partial\rho)\Pi = -f(i)y(i)\sigma_c\sigma_p\psi(y(i)P - f(i)C)$. Further, $\lim_{i\to 1}(\partial^2/\partial i\partial\rho)\Pi = \tau_\rho(m)\psi(P - C)\sigma_c\sigma_p/s^4$, where

$$\tau_\rho(m) = \lambda(m)y'(1) + \theta(m)f'(1).$$

Note that $(\partial^2/\partial m^2)\tau_\rho(m) = 2\sigma_c(\sigma_c - \rho\sigma_p)(y'(1) - f'(1)) > 0$ and $\lim_{m\to 0}(\partial/\partial m)\tau_\rho(m) = \mu_c s^2(y'(1) - f'(1)) > 0$. Therefore, $\tau_p(m, \rho)$ is convex increasing in m on $m \geq 0$. Further, $\lim_{m\to\infty}\tau_p(m) > 0$. Thus, $\tau_p(m) > 0$ and hence (15) if and only if $m > \underline{m}$. Note that $\underline{m} < 0$ if $\tau_p(0) > 0$. (Note that because $\tau_p(m)$ is quadratic in m and $\overline{m}$ is the larger root of $\tau_p(m) = 0$, $\overline{m}$ is readily available in closed form.)■

Proof of Proposition 3: First, we establish the results regarding (16). Suppose $\sigma_p = 0$. Then $(\partial/\partial\sigma_c)\Delta = f_c(m)/\sqrt{2\pi}$ where $f_c(m) = (k/i)e^{-(m+(i-1)\mu_p)^2/(2\sigma_c^2)} + (1 - k/i)e^{-(m-(1-j)\mu_p)^2/(2\sigma_c^2)} - e^{-m^2/(2\sigma_c^2)}$. If $m \leq \mu_c(1 - j)/(1 + j)$, then $m \leq \left|m - (1 - j)\mu_p\right|$, which implies $f_c(m) < 0$. Because $f_c(\cdot)$ is continuous, if $f_c(m) = 0$ has no positive root, then the result regarding (16) holds with $\overline{m} = \infty$. Suppose instead that $f_c(m) = 0$ has at least one positive root. We will show that only one root exists. Let $\tilde{m}_n^c$ denote a root $f_c(m) = 0$, and index the roots so that $0 < \tilde{m}_1^c < \tilde{m}_2^c < ...$ Then $(\partial/\partial m)f_c(\tilde{m}_n^c) = g_c(\tilde{m}_n^c)\mu_p(i-1)k/(i\sigma_c^2)$, where $g_c(m) = e^{-(m-(1-j)\mu_p)^2/(2\sigma_c^2)} - e^{-(m+(i-1)\mu_p)^2/(2\sigma_c^2)}$. Note that $g_c(m) \geq 0$ if and only if $m \geq \mu_p(i-1)(2k-i)/[2(i-k)]$, where the first inequality is strict if and only if the second is strict. Because $f_c(m) < 0$ for $m \leq \mu_c(1-j)/(1+j)$ and $f_c(\cdot)$ is continuous, it must be that $(\partial/\partial m)f_c(\tilde{m}_1^c) > 0$, which implies $\tilde{m}_1^c > \mu_p(i - 1)(2k - i)/[2(i - k)]$. Suppose there exists a second root $\tilde{m}_2^c$. Then because $\tilde{m}_2^c > \tilde{m}_1^c > \mu_p(i-1)(2k-i)/[2(i-k)]$, $(\partial/\partial m)f_c(\tilde{m}_2^c) > 0$, a contradiction because $(\partial/\partial m)f_c(\tilde{m}_1^c) > 0$ and $f_c(\cdot)$ is continuous. Therefore, $f_c(m) = 0$ has one root, and $\overline{m} = \tilde{m}_1^c$. Second, we establish the results regarding (17). Suppose $\sigma_c = 0$. Then $(\partial/\partial\sigma_p)\Delta = f_p(m)/\sqrt{2\pi}$ where $f_p(m) = ke^{-(m+(1-1/i)\mu_c)^2/(2\sigma_p^2)} + (1-k)e^{-(m-(1/j-1)\mu_c)^2/(2\sigma_p^2)} - e^{-m^2/(2\sigma_p^2)}$. If $m \leq \mu_c(1-j)/(2j)$, then $m \leq |m-(1/j-1)\mu_c|$, which implies $f_p(m) < 0$. The remainder of the proof follows by argument parallel to that above, where $\tilde{m}_n^p$ is defined analogously to $\tilde{m}_n^c$ and $g_p(m) = e^{-(m-(1/j-1)\mu_c)^2/(2\sigma_c^2)} - e^{-(m+(1-1/i)\mu_c)^2/(2\sigma_c^2)}$ is defined analogously to $g_c(m)$. Then $(\partial/\partial m)f_p(\tilde{m}_n^p) = g_p(\tilde{m}_n^p)\mu_c(i-1)k/(i\sigma_p^2)$; further, $g_p(m) \geq 0$ if and only if $m \geq \mu_c(2k-1)(i-1)/[2i(1-k)]$, where the first inequality is strict if and only if the second is strict.■

Internet Appendix

This appendix generalizes Proposition 2 by extending the result to allow for correlation $\rho \in (-1, 1)$. The results are easily extended to $\rho \in \{1, -1\}$, although the statements of the results are slightly more complex. This appendix provides results, in turn, for the value of capacity-efficiency, the value of input-efficiency when input-efficiency is improved by increasing the output yield, and the value of input-efficiency when input-efficiency is improved by decreasing the input required per unit output.

Value of Capacity-Efficiency

Proposition 4 *The value of capacity-efficiency is increasing in input cost variability*

$$(\partial^2/\partial k \partial \sigma_c)\Pi \geq 0 \tag{23}$$

if and only if the correlation is small $\rho \leq \sigma_c/\sigma_p$*, where the inequality in* (23) *is strict if and only if* $\rho < \sigma_c/\sigma_p$*. The value of capacity-efficiency is increasing in output price variability*

$$(\partial^2/\partial k \partial \sigma_p)\Pi \geq 0 \tag{24}$$

if and only if the correlation is small $\rho \leq \sigma_p/\sigma_c$*, where the inequality in* (24) *is strict if and only if* $\rho < \sigma_p/\sigma_c$*. The value of capacity-efficiency is decreasing the correlation between the input cost and the output price*

$$(\partial^2/\partial k \partial \rho)\Pi < 0. \tag{25}$$

Proof of Proposition 4: For (2), observe that $\lim_{k\to 1}(\partial^2/\partial k \partial \sigma_c)\Pi = (\sigma_c - \rho\sigma_p)\psi(P - C)$, $\lim_{k\to 1}(\partial^2/\partial k \partial \sigma_p)\Pi = (\sigma_p - \rho\sigma_c)\psi(P - C)$ and $\lim_{k\to 1}(\partial^2/\partial k \partial \rho)\Pi = -\sigma_c\sigma_p\psi(P - C)$.■

Value of Input-Efficiency When Input-Efficiency is Improved by Increasing the Output Yield

In this subsection, suppose that input-efficiency is improved by increasing the output yield as in (3).

Proposition 5 *If the correlation is small $\rho \leq \sigma_c/\sigma_p$, then the value of input-efficiency is decreasing in input cost variability*

$$(\partial^2/\partial i \partial \sigma_c)\Pi \leq 0, \tag{26}$$

where the inequality is strict if and only if $\rho < \sigma_c/\sigma_p$ or $\sigma_c \neq \sigma_p$. If the correlation is large $\rho > \sigma_c/\sigma_p$, then the value of input-efficiency is increasing in output price variability

$$(\partial^2/\partial i \partial \sigma_c)\Pi > 0 \tag{27}$$

if and only if the expected input cost is large $\mu_c > \underline{\mu}_c$ and the expected margin is moderate $m \in (\underline{m}, \overline{m})$, where $0 < \underline{m} < \overline{m} < \infty$.

Proof of Proposition 5: For (3), observe that $(\partial/\partial\sigma_c)\Pi = (\sigma_c - i\rho\sigma_p)\psi(iP - C)$. Further, $\lim_{i\to 1}(\partial^2/\partial i\partial\sigma_c)\Pi = \tau_c(m)\psi(P - C)/s^4$, where

$$\tau_c(m) = \alpha(\mu_c, \mu_c + m, \sigma_c, \sigma_p, \rho).$$

If $\rho = \sigma_c/\sigma_p$, then $\tau_c(m) = -\sigma_c(\sigma_c^2 - \sigma_p^2)^2$, which implies (26), where the inequality is strict if and only if $\sigma_c \neq \sigma_p$. Suppose for the remainder of the proof that $\rho \neq \sigma_c/\sigma_p$. This implies $(\partial^2/\partial m^2)\tau_c(m) = -2\sigma_c(\sigma_c - \rho\sigma_p)^2 < 0$. Let $\underline{\mu}_c \equiv 2\sqrt{1-\rho^2}\sigma_c\sigma_p/s$. If $\mu_c < \underline{\mu}_c$, then $\tau_c(m) < 0$ for $m \geq 0$, which implies that (26) holds with strict inequality. If $\mu_c \geq \underline{\mu}_c$, then $\tau_c(m) = 0$ has two roots

$$m_1 = s\left(\mu_c s - \sqrt{\mu_c^2 s^2 - 4(1-\rho^2)\sigma_c^2\sigma_p^2}\right) \Big/ [2\sigma_c(\rho\sigma_p - \sigma_c)]$$
$$m_2 = s\left(\mu_c s + \sqrt{\mu_c^2 s^2 - 4(1-\rho^2)\sigma_c^2\sigma_p^2}\right) \Big/ [2\sigma_c(\rho\sigma_p - \sigma_c)],$$

where the roots are distinct if and only if $\mu_c > \underline{\mu}_c$; further, because $\tau_c(m)$ is concave in m, $\tau_c(m) > 0$ if and only if $m \in (m_1, m_2)$.

First, suppose $\rho < \sigma_c/\sigma_p$. If $\mu_c < \underline{\mu}_c$, then (26) holds with strict inequality. If $\mu_c \geq \underline{\mu}_c$, then $0 > m_1 \geq m_2$. This implies that for $m \geq 0$, $\tau_c(m) < 0$, which in turn implies that (26) holds with strict inequality. Second, suppose $\rho > \sigma_c/\sigma_p$. If $\mu_c \leq \underline{\mu}_c$, then (26) holds. If $\mu_c > \underline{\mu}_c$, then $0 < m_1 < m_2 < \infty$. Because $\tau_c(m) > 0$ if and only if $m \in (m_1, m_2)$, the result holds with $\underline{m} = m_1$ and $\overline{m} = m_2$.■

Proposition 6 *If the correlation is small $\rho < \sigma_p/\sigma_c$, then the value of input-efficiency is decreasing in output price variability*

$$(\partial^2/\partial i \partial \sigma_p)\Pi < 0 \tag{28}$$

if and only if the one of the following holds: the correlation is small $\rho \leq \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$ and the expected margin is large $m > \overline{m}$, where $\overline{m} > 0$; or the correlation is moderate $\rho \in (\sigma_c/\sigma_p, \sigma_p/\sigma_c)$ and the expected margin is moderate $m \in (\underline{m}, \overline{m})$, where $\underline{m} \leq \overline{m} < \infty$.

If the correlation is large $\rho > \sigma_p/\sigma_c$, then the value of input-efficiency is increasing in output price variability

$$(\partial^2/\partial i \partial \sigma_p)\Pi > 0 \tag{29}$$

if and only if the expected margin is large $m > \overline{m}$ or the correlation is moderate $\rho \in (\sigma_p/\sigma_c, \overline{\rho})$, where $\overline{m} \in [0, \infty)$ and $\overline{\rho} \in (\sigma_p/\sigma_c, 1)$. If $\rho = \sigma_p/\sigma_c$, then (29) holds.

Proof of Proposition 6: For (3), observe that $(\partial/\partial\sigma_p)\Pi = i(i\sigma_p - \rho\sigma_c)\psi(iP - C)$. Further, $\lim_{i\to 1}(\partial^2/\partial i\partial\sigma_p)\Pi = \tau_p(m)\psi(P - C)/s^4$, where

$$\tau_p(m) = \beta(\mu_c + m, \mu_c, \sigma_p, \sigma_c).$$

Observe that

$$(\partial^2/\partial m^2)\tau_p(m) = 2\sigma_c[\rho(\sigma_c^2 + \sigma_p^2) - (1 + \rho^2)\sigma_c\sigma_p], \tag{30}$$

which has the same sign as $\rho - \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$. If this quantity is non-zero, then $(\partial/\partial m)\tau_p(m) = 0$ has a unique solution in m,

$$\hat{m} = \mu_c(\sigma_c^2 - 2\rho\sigma_c\sigma_p + \sigma_p^2)/[2\sigma_c(\rho\sigma_p - \sigma_c)]. \tag{31}$$

Note that $\tau_p(0) = s^2\gamma(\sigma_p, \sigma_c)$. A technical result that will be useful subsequently is that $\tau_p(0) > 0$ if and only if $\rho < \underline{\rho}(\sigma_p, \sigma_c)$; further, if $\sigma_c > \sigma_p$, then $\underline{\rho}(\sigma_p, \sigma_c) \in (\sigma_p/\sigma_c, 1)$; otherwise, $\underline{\rho}(\sigma_p, \sigma_c) \geq 1$. First, suppose that $\rho > \sigma_p/\sigma_c$. This implies that $\sigma_c > \sigma_p$, (30) is strictly positive, and (31) is strictly negative. Therefore, $\tau_p(m)$ is convex increasing in m for $m \geq 0$. If $\rho \in (\sigma_p/\sigma_c, \underline{\rho}(\sigma_p, \sigma_c))$, then $0 < \tau_p(0) \leq \tau_p(m)$ for $m \geq 0$, which implies (29). If $\rho \geq \underline{\rho}(\sigma_p, \sigma_c)$, then $\tau_p(0) \leq 0$. Because

$\tau_p(\cdot)$ is increasing and $\lim_{m\to\infty}\tau_p(m) > 0$, (29) holds if and only if $m > \overline{m}$ for some threshold $\overline{m} \in [0, \infty)$. The result holds with $\overline{\rho} = \underline{\rho}(\sigma_p, \sigma_c)$.

Second, suppose that $\sigma_c > \sigma_p$ and $\rho = \sigma_p/\sigma_c$. This implies $\tau_p(m) = \sigma_p(\sigma_c^2 - \sigma_p^2)^2 > 0$.

Third, suppose that $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$. This implies (30) is strictly negative, and (31) is strictly negative. Therefore, $\tau_p(m)$ is concave decreasing in m for $m \geq 0$. Further, $\lim_{m\to\infty}\tau_p(m) < 0$. Because $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c) \leq \underline{\rho}(\sigma_p, \sigma_c)$, $\tau_p(0) > 0$. Therefore, $\tau_p(m) < 0$, or equivalently (28), if and only if $m > \overline{m}$ for some threshold $\overline{m} \in (0, \infty)$.

Fourth, suppose that $\sigma_c < \sigma_p$ and $\rho = \sigma_c/\sigma_p$. This implies $\tau_p(m) = (\sigma_p^2 - m\mu_c)(\sigma_c^2 - \sigma_p^2)^2/\sigma_p$, so $(\partial^2/\partial i \partial \sigma_p)\Pi < 0$ if and only if $m > \sigma_p^2/\mu_c$.

Fifth, suppose that $\sigma_c < \sigma_p$ and $\rho \in (\sigma_c/\sigma_p, \sigma_p/\sigma_c)$. This implies that (30) is strictly positive, and (31) is strictly positive. Therefore, $\tau_p(m)$ is convex in m for $m \geq 0$. Further, $\sigma_c < \sigma_p$ implies $\tau_p(0) > 0$. Also, $\lim_{m\to\infty}\tau_p(m) > 0$. Therefore (28) holds if and only if $m \in (\underline{m}, \overline{m})$, where $\underline{m} \leq \overline{m} < \infty$.■

Proposition 7 *The value of input-efficiency is increasing in the correlation*

$$(\partial^2/\partial i \partial \rho)\Pi > 0 \tag{32}$$

if and only if one of the following holds: the correlation is large $\rho > \sigma_c/\sigma_p$ and the expected margin is small $m < \overline{m}$, where $\overline{m} \in (0, \infty)$; the correlation is small $\rho < \sigma_c/\sigma_p$ and the expected margin is large $m > \overline{m}$, where $\overline{m} \in (0, \infty)$; or $\rho = \sigma_c/\sigma_p$, and $m > 0$.

Proof of Proposition 7: For (3), observe that $\lim_{i\to 1}(\partial^2/\partial i \partial \rho)\Pi = \tau_\rho(m)\psi(P - C)\sigma_c\sigma_p/s^4$, where

$$\tau_\rho(m) = \lambda(m).$$

If $\rho = \sigma_c/\sigma_p$, then $\tau_\rho(m) = \mu_c m(1 - \rho^2)\sigma_p^2$; thus, $\tau_\rho(m) > 0$ and equivalently (32) holds if and only if $m > 0$. Suppose instead that $\rho \neq \sigma_c/\sigma_p$. Then, $\tau_\rho(m) = 0$ has two roots:

$$m_1 = s\left(\mu_c s - \sqrt{\mu_c^2 s^2 + 4\sigma_c^2(\rho\sigma_p - \sigma_c)^2}\right) / \left[2\sigma_c(\rho\sigma_p - \sigma_c)\right],$$

$$m_2 = s\left(\mu_c s + \sqrt{\mu_c^2 s^2 + 4\sigma_c^2(\rho\sigma_p - \sigma_c)^2}\right) / \left[2\sigma_c(\rho\sigma_p - \sigma_c)\right].$$

Further, $(\partial^2/\partial m^2)\tau_\rho(m) = 2\sigma_c(\sigma_c - \rho\sigma_p)$. If $\rho > \sigma_c/\sigma_p$, then $\tau_\rho(m)$ is concave in m. Further, $m_1 < 0 < m_2 < \infty$; the result follows with $\overline{m} = m_2$. If $\rho < \sigma_c/\sigma_p$, then $\tau_\rho(m)$ is convex in m. Further, $m_2 < 0 < m_1 < \infty$; the result follows with $\overline{m} = m_1$.■

Value of Input-Efficiency When Input-Efficiency is Improved by Decreasing the Input Required Per Unit Output

In this subsection, suppose that input-efficiency is improved by decreasing the input required per unit output as in (4).

Proposition 8 *If the correlation is small $\rho < \sigma_c/\sigma_p$, then the value of input-efficiency is decreasing in feedstock cost variability*

$$(\partial^2/\partial i\partial\sigma_c)\Pi < 0. \tag{33}$$

If the correlation is large $\rho > \sigma_c/\sigma_p$, then the value of input-efficiency is increasing in input cost variability

$$(\partial^2/\partial i\partial\sigma_c)\Pi > 0 \tag{34}$$

if and only if the expected input cost is large $\mu_c > \underline{\mu}_c$ and the expected margin is moderate $m \in (\underline{m}, \overline{m})$, where $\underline{m} < \overline{m} < \infty$; further, $\underline{m} > 0$ if and only if $\rho < \overline{\rho}$, where $\overline{\rho} > 0$. If $\rho = \sigma_c/\sigma_p$, then $(\partial^2/\partial i\partial\sigma_c)\Pi \leq 0$, where the inequality is strict if and only if $\sigma_c \neq \sigma_p$.

Proof of Proposition 8: For (4), observe that $(\partial/\partial\sigma_c)\Pi = (\sigma_c/i - \rho\sigma_p)\psi(P - C/i)/i$. Further, $\lim_{i\to 1}(\partial^2/\partial i\partial\sigma_c)\Pi = \tau_c(m)\psi(P - C)/s^4$, where

$$\tau_c(m) = -\beta(\mu_c, \mu_c + m, \sigma_c, \sigma_p).$$

If $\rho = \sigma_c/\sigma_p$, then $\tau_c(m) = -\sigma_c(\sigma_c^2 - \sigma_p^2)^2 \leq 0$, where the inequality is strict if and only if $\sigma_c \neq \sigma_p$. Suppose for the remainder of the proof that $\rho \neq \sigma_c/\sigma_p$. Because $(\partial^2/\partial m^2)\tau_c(m) = -2\sigma_c(\sigma_c - \rho\sigma_p)^2 < 0$, $\tau_c(m)$ is concave in m. Let $\underline{\mu}_c \equiv 2\sqrt{\sigma_c \max(\gamma(\sigma_c, \sigma_p), 0)}/s$. If $\mu_c < \underline{\mu}_c$, then

$\tau_c(m) < 0$ for $m \geq 0$, which implies (33). If $\mu_c \geq \underline{\mu}_c$, then $\tau_c(m) = 0$ has two roots

$$m_1 = s\left(\mu_c s - \sqrt{\mu_c^2 s^2 - 4\sigma_c \gamma(\sigma_c, \sigma_p)}\right) \Big/ \left[2\sigma_c(\rho\sigma_p - \sigma_c)\right]$$
$$m_2 = s\left(\mu_c s + \sqrt{\mu_c^2 s^2 - 4\sigma_c \gamma(\sigma_c, \sigma_p)}\right) \Big/ \left[2\sigma_c(\rho\sigma_p - \sigma_c)\right],$$

where the roots are distinct if and only if $\mu_c > \underline{\mu}_c$ or $\gamma(\sigma_c, \sigma_p) < 0$; further, $\tau_c(m) < 0$ if and only if $m \in (m_1, m_2)$. It is straightforward to show that $\gamma(\sigma_c, \sigma_p) > 0$ holds if and only if $\rho < \underline{\rho}(\sigma_c, \sigma_p)$. Further, if $\sigma_c < \sigma_p$, then $\underline{\rho}(\sigma_c, \sigma_p) \in (\sigma_c/\sigma_p, 1)$; otherwise $\rho_c \geq 1$.

First, suppose $\rho < \sigma_c/\sigma_p$. This implies $\rho < \underline{\rho}(\sigma_c, \sigma_p)$ and hence $\gamma(\sigma_c, \sigma_p) > 0$. If $\mu_c < \underline{\mu}_c$, then (33) holds. If $\mu_c \geq \underline{\mu}_c$, then $0 > m_1 \geq m_2$. This implies that for $m \geq 0$, $\tau_c(m) < 0$, which in turn implies that (33) holds.

Second, suppose $\rho > \sigma_c/\sigma_p$. If $\mu_c \leq \underline{\mu}_c$, then $(\partial^2/\partial i \partial \sigma_c)\Pi \leq 0$. If $\mu_c > \underline{\mu}_c$, then $\max(0, m_1) < m_2$. Further, $m_1 > 0$ if and only if $\rho < \underline{\rho}(\sigma_c, \sigma_p)$. Because $\tau_c(m) < 0$ if and only if $m \in (m_1, m_2)$, the result holds with $\underline{m} = m_1$, $\overline{m} = m_2$ and $\overline{\rho} = \underline{\rho}(\sigma_c, \sigma_p)$.■

Proposition 9 *If the correlation is small $\rho < \sigma_p/\sigma_c$, then the value of input-efficiency is decreasing in input cost variability*

$$(\partial^2/\partial i \partial \sigma_p)\Pi < 0 \tag{35}$$

if and only if one of the following holds: the correlation is small $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$ and the expected margin is large $m > \overline{m}$, where $\overline{m} \in (0, \infty)$; or the correlation is moderate $\rho \in (\sigma_c/\sigma_p, \sigma_p/\sigma_c)$, the expected input cost is large $\mu_c > \underline{\mu}_c$ where $\underline{\mu}_c \in (0, \infty)$ and the expected margin is moderate $m \in (\underline{m}, \overline{m})$, where $0 < \underline{m} < \overline{m} < \infty$.

If the correlation is large $\rho > \sigma_p/\sigma_c$, then the value of input-efficiency is increasing in output price variability

$$(\partial^2/\partial i \partial \sigma_p)\Pi > 0. \tag{36}$$

If $\rho = \sigma_p/\sigma_c$, then $(\partial^2/\partial i \partial \sigma_c)\Pi \geq 0$, where the inequality is strict if and only if $\sigma_c \neq \sigma_p$.

Proof of Proposition 9: For (4), observe that $(\partial/\partial\sigma_p)\Pi = (\sigma_p - \rho\sigma_c/i)\psi(P - C/i)$. Further, $\lim_{i\to 1}(\partial^2/\partial i\partial\sigma_p)\Pi = \tau_p(m)\psi(P - C)/s^4$, where

$$\tau_p(m) = -\alpha(\mu_c + m, \mu_c, \sigma_p, \sigma_c).$$

If $\rho = \sigma_p/\sigma_c$, then $\tau_p(m) = \sigma_p(\sigma_c^2 - \sigma_p^2)^2 \geq 0$, where the inequality is strict if and only if $\sigma_c \neq \sigma_p$. Suppose for the remainder of the proof that $\rho \neq \sigma_c/\sigma_p$. Observe that

$$(\partial^2/\partial m^2)\tau_p(m) = 2\sigma_c[\rho(\sigma_c^2 + \sigma_p^2) - (1 + \rho^2)\sigma_c\sigma_p], \tag{37}$$

which has the same sign as $\rho - \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$. If this quantity is non-zero, then $(\partial/\partial m)\tau_p(m) = 0$ has a unique solution in m,

$$\hat{m} = \mu_c s^2/[2\sigma_c(\rho\sigma_p - \sigma_c)]. \tag{38}$$

Further, $\tau_p(0) = (1 - \rho^2)\sigma_c^2\sigma_p s^2 > 0$.

First, suppose that $\rho > \sigma_p/\sigma_c$. This implies that $\sigma_c > \sigma_p$, (37) is strictly positive, and (38) is strictly negative. Therefore, $\tau_p(m)$ is convex increasing in m for $m \geq 0$. Therefore, for $m \geq 0$, $\tau_p(m) \geq \tau_p(0) > 0$, which implies (36).

Second, suppose that $\rho < \min(\sigma_p/\sigma_c, \sigma_p/\sigma_c)$. This implies (37) is strictly negative, and (38) is strictly negative. Therefore, $\tau_p(m)$ is concave decreasing in m for $m \geq 0$. Because $\tau_p(0) > 0$, $\tau_p(\cdot)$ is decreasing, and $\lim_{m\to\infty} \tau_p(m) < 0$. Therefore (35) holds if and only if $m > \overline{m}$, where $\overline{m} \in (0, \infty)$.

Third, suppose that $\sigma_c < \sigma_p$ and $\rho \in (\sigma_c/\sigma_p, \sigma_p/\sigma_c)$. This implies that (37) is strictly positive, and (38) is strictly positive. Therefore, $\tau_p(m)$ is convex in m for $m \geq 0$. Further, $\lim_{m\to\infty} \tau_p(m) > 0$. With a little effort one can show that $\tau_p(\hat{m}) < 0$ if and only if $\mu_c > \underline{\mu}_c$, where

$$\underline{\mu}_c = 2\sqrt{(1 - \rho^2)\sigma_c^3\sigma_p(\rho\sigma_p - \sigma_c)/[2\rho^2\sigma_c^2\sigma_p - \rho\sigma_c(\sigma_c^2 + 3\sigma_p^2) + \sigma_p(\sigma_c^2 + \sigma_p^2)]}.$$

Therefore, if $\mu_c > \underline{\mu}_c$, then (35) for $m \in (\underline{m}, \overline{m})$, where $0 < \underline{m} < \overline{m} < \infty$, where the first inequality follows because $\tau_p(0) > 0$. If $\mu_c < \underline{\mu}_c$, then for $m \geq 0$, $\tau_p(m) \geq \tau_p(\hat{m}) \geq 0$, so (35) is violated.■

Proposition 10 *The value of input-efficiency is increasing in the correlation*

$$(\partial^2/\partial i\partial\rho)\Pi > 0 \tag{39}$$

if and only if one of the following holds: the correlation is small $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$; the correlation is large $\rho > \sigma_p/\sigma_c$ and the expected margin is large $m > \overline{m}$, where $\overline{m} \in (0, \infty)$; the correlation is large $\rho > \sigma_c/\sigma_p$ and the expected margin is small $m < \overline{m}$, where $\overline{m} > 0$; or $\rho = \sigma_c/\sigma_p$.

Proof of Proposition 10:

For (4), observe that $(\partial/\partial\rho)\Pi = -\sigma_c\sigma_p\psi(P - C/i)/i$. Further, $\lim_{i\to 1}(\partial^2/\partial i\partial\rho)\Pi = \tau_\rho(m)\psi(P - C)\sigma_c\sigma_p/s^4$, where

$$\tau_\rho(m) = -\theta(m).$$

If $\rho = \sigma_c/\sigma_p$, then $\tau_\rho(m) = (1-\rho^2)\sigma_p^2(\mu_c m + (1-\rho^2)\sigma_p^2) > 0$. Suppose instead that $\rho \neq \sigma_c/\sigma_p$. Note that $(\partial^2/\partial m^2)\tau_\rho(m) = 2\sigma_c(\sigma_c - \rho\sigma_p)$. Let $\underline{\mu}_c = 2\sqrt{\sigma_c\sigma_p[(1+\rho^2)\sigma_c\sigma_p - \rho(\sigma_c^2+\sigma_p^2)]}/s$. If $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$ and $\mu_c \leq \underline{\mu}_c$, then $\tau_\rho(m) > 0$ for $m \geq 0$; otherwise, $\tau_\rho(m) = 0$ has two roots:

$$m_1 = s\left(\mu_c s - \sqrt{\mu_c^2 s^2 - 4\sigma_c\sigma_p[(1+\rho^2)\sigma_c\sigma_p - \rho(\sigma_c^2+\sigma_p^2)]}\right) / \left[2\sigma_c(\rho\sigma_p - \sigma_c)\right],$$

$$m_2 = s\left(\mu_c s + \sqrt{\mu_c^2 s^2 - 4\sigma_c\sigma_p[(1+\rho^2)\sigma_c\sigma_p - \rho(\sigma_c^2+\sigma_p^2)]}\right) / \left[2\sigma_c(\rho\sigma_p - \sigma_c)\right].$$

If $\rho < \min(\sigma_c/\sigma_p, \sigma_p/\sigma_c)$ and $\mu_c > \underline{\mu}_c$, then $\tau_\rho(m)$ is convex in m. Further, $m_2 < m_1 < 0$, so $\tau_\rho(m) > 0$ for $m \geq 0$. Similarly, if $\rho > \sigma_p/\sigma_c$, then $\tau_\rho(m)$ is convex in m, $m_2 < 0 < m_1 < \infty$, and the result follows with $\overline{m} = m_1$. If $\rho > \sigma_c/\sigma_p$, then $\tau_\rho(m)$ is concave in m. Further, $m_1 < 0 < m_2 < \infty$; the result follows with $\overline{m} = m_2$.■